1. Prove the Cauchy inequality

\[(\sum_{i=1}^{n} x_i y_i)^2 \leq \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} y_i^2\]

using the Einstein convention.

**Solution:** We discussed the inequality in the previous video. Here we use the Einstein convention to give a "better" proof.

The left-hand side, by the Einstein Convention, can be written as

\[(\sum_{i=1}^{n} x_i y_i)^2 = \sum_{i=1}^{n} x_i y_i \sum_{j=1}^{n} x_j y_j = x_i y_i x_j y_j\]

The right-hand side is

\[\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2 = \sum_{i=1}^{n} x_i^2 \sum_{j=1}^{n} y_j^2 = x_i^2 y_j^2\]

\[LHS - RHS = x_i^2 y_j^2 - x_i x_j y_i y_j = \frac{1}{2} (x_i y_j - x_j y_i)^2 \geq 0\]