MATH 2A/5A Prep: Exponents and Radicals

Facts to Know:

Suppose a, b, c are positive numbers, m, n are positive integers, then

- $a^{b+c} = a^b \cdot a^c \neq a^b + a^c$ e.g. $a^{x+1} = a \cdot a^x \neq a + a^x$
- $a^{bc} = (a^b)^c \neq a^b \cdot a^c$ e.g. $a^{x^2} = a^{(x^2)} = a^{x \cdot x} = (a^x)^x \neq a^x \cdot a^x = a^{2x}$
- $(a+b)^n \neq a^n + b^n$ e.g. If $n=2$, $(a+b)^2 = a^2 + 2ab + b^2 \neq a^2 + b^2$
- $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ e.g. $\sqrt{1+x^2} \neq \sqrt{1} + \sqrt{x^2} = 1 + x^2$
- $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ e.g. $\sqrt{2} \cdot \sqrt{18} = \sqrt{2 \cdot 18} = \sqrt{36} = 6$ (reduced)
- $a^{1/n} = \sqrt[n]{a}$ e.g. $2^{\frac{1}{2}} = \sqrt{2}$, $2^{\frac{1}{3}} = \sqrt[3]{2}$
- $a^{-1} = \frac{1}{a}$ $a^{-m} = a^{m \cdot (-1)} = (a^m)^{(-1)} = \frac{1}{a^m}$
- $a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$ $a^{-\frac{m}{n}} = a^{m \cdot (\frac{1}{n}) \cdot (-1)} = \left[(a^m)^{\frac{1}{n}}\right]^{(-1)} = \frac{1}{(a^m)^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{a^m}}$

Examples:

1. Suppose $x > 0$. Simplify the expression $\left[(x^2-1)^2 + (2x^2-1)\right]^{-\frac{3}{4}}$.