MATH 2A/5A Prep: Exponents and Radicals

1. Expand the expression $(\sqrt{x}+1)^3$

Solution:
Method 1:

$$\begin{aligned}(\sqrt{x}+1)^3 &= (\sqrt{x}+1)(\sqrt{x}+1)(\sqrt{x}+1)\\ &= (\sqrt{x}\cdot\sqrt{x}+\sqrt{x}+\sqrt{x}+1)(\sqrt{x}+1)\\ &= (x+2\sqrt{x}+1)(\sqrt{x}+1)\\ &= x\sqrt{x}+2\sqrt{x}\cdot\sqrt{x}+\sqrt{x}+x+2\sqrt{x}+1\\ &= x\sqrt{x}+3x+3\sqrt{x}+1\end{aligned}$$

Method 2: If you know the binomial formula, you can use it to get

$$\begin{aligned}(\sqrt{x}+1)^3 &= (\sqrt{x})^3+3(\sqrt{x})^2+3\sqrt{x}+1\\ &= x\sqrt{x}+3x+3\sqrt{x}+1\end{aligned}$$

2. Function $f(x)$ is defined by $f(x)=2^{x^2}\cdot 4^{-x}$, find $f(3)$.

Solution:

$$\begin{aligned}f(3)=2^{3^2}\cdot 4^{-3} &= 2^9\cdot 4^{-3}\\ &= 2^9\cdot(2^2)^{-3}\\ &= 2^9\cdot 2^{2\cdot(-3)}\\ &= 2^9\cdot 2^{-6}\\ &= 2^{9-6}\\ &= 2^3 = 8\end{aligned}$$

3. Simplify the expression $\dfrac{2^{x^2}}{4^x}$.

Solution:

$$\begin{aligned}\frac{2^{x^2}}{4^x} &= 2^{x^2}\cdot 4^{-x}\\ &= 2^{x^2}\cdot(2^2)^{-x}\\ &= 2^{x^2}\cdot 2^{2\cdot(-x)}\\ &= 2^{x^2}\cdot 2^{-2x}\\ &= 2^{x^2-2x}\end{aligned}$$

4. Simplify the expression $(\sqrt{x+1}-\sqrt{x})^2$

Solution:

$$\begin{aligned}(\sqrt{x+1}-\sqrt{x})^2 &= (\sqrt{x+1})^2 - 2\sqrt{x}\cdot\sqrt{x+1} + (\sqrt{x})^2 \\ &= x+1-2\sqrt{x(x+1)}+x \\ &= 2x+1-2\sqrt{x^2+x}\end{aligned}$$

5. Simplify the number $9^{\frac{1}{4}}\sqrt{6}-\sqrt{8}$

Solution:

$$\begin{aligned}9^{\frac{1}{4}}\sqrt{6}-\sqrt{8} &= (3^2)^{\frac{1}{4}}\sqrt{6}-\sqrt{4\cdot 2} \\ &= 3^{2\cdot\frac{1}{4}}\sqrt{6}-\sqrt{4}\cdot\sqrt{2} \\ &= 3^{\frac{1}{2}}\sqrt{6}-2\sqrt{2} \\ &= \sqrt{3}\cdot\sqrt{6}-2\sqrt{2} \\ &= \sqrt{3}\cdot\sqrt{3}\cdot\sqrt{2}-2\sqrt{2} \\ &= 3\sqrt{2}-2\sqrt{2} \\ &= \sqrt{2}\end{aligned}$$