MATH 2A/5A Prep: Quadratic functions

1. Find the two intersection points of the parabola $y = 2x^2 + x - 1$ with the line $y = -2x + 2$

Solution: If (x, y) is an intersection point, then x and y satisfy the equations

$$\begin{cases} y = 2x^2 + x - 1 \\ y = -2x + 2 \end{cases}$$

So

$$2x^2 + x - 1 = y = -2x + 2$$

So

$$2x^2 + 3x - 3 = 0$$

The discriminant is $\Delta = 3^2 - 4 \cdot 2 \cdot (-3) = 33 > 0$, so two solutions for x are

$$x = \frac{-3 \pm \sqrt{33}}{4}$$

If $x = \dfrac{-3+\sqrt{33}}{4}$, we have $y = -2x + 2 = \dfrac{7-\sqrt{33}}{2}$.
If $x = \dfrac{-3-\sqrt{33}}{4}$, we have $y = -2x + 2 = \dfrac{7+\sqrt{33}}{2}$.
So two intersection points are

$$\left(\frac{-3+\sqrt{33}}{4}, \frac{7-\sqrt{33}}{2}\right) \text{ and } \left(\frac{-3-\sqrt{33}}{4}, \frac{7+\sqrt{33}}{2}\right)$$

2. Solve the equation $2x^2 - 9x + 10 = 0$

Solution: By the cross-multiplication method,

$$\begin{array}{cc} 2x & -5 \\ x & -2 \\ \hline \multicolumn{2}{c}{2x \cdot (-2) + x \cdot (-5) = -9x} \end{array}$$

So

$$2x^2 - 9x + 10 = 0$$

is same as

$$(2x - 5)(x - 2) = 0.$$

So $2x - 5 = 0$ or $x - 2 = 0$. So $x = \frac{5}{2}$ or $x = 2$.

3. Use the quadratic formula to explain why $x^2 - 6x + 9 = 0$ has a unique solution.

Solution: The determinant is

$$\Delta = 6^2 - 4 \cdot 9 \cdot 1 = 0$$

So the solution is

$$x = \frac{-(-6) \pm \sqrt{0}}{2}$$

The two solutions given by this formula are $x = 3 + 0$ and $x = 3 - 0$, they are the same solution. So the equation has a unique solution $x = 3$.