MATH 2A/5A Prep: Trigonometric Functions

1. Express $\cos^2(x)$ in terms of $\cos(2x)$.

Solution:

$$\begin{aligned} 2\cos^2(x) - 1 &= \cos(2x) \\ 2\cos^2(x) &= 1 + \cos(2x) \\ \cos^2(x) &= \frac{1+\cos(2x)}{2} \end{aligned}$$

2. Express $\sin^4(x)$ in the form $a + b\cos(2x) + c\cos(4x)$, where a, b, c are real numbers.

Solution:

$$\begin{aligned} \sin^4(x) &= [\sin^2(x)]^2 \\ &= \left[\frac{1-\cos(2x)}{2}\right]^2 \\ &= \frac{1 - 2\cos(2x) + \cos^2(2x)}{4} \\ &= \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x) \\ &= \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\left[\frac{1+\cos(4x)}{2}\right] \\ &= \frac{3}{8} - \frac{1}{2}\cos(2x) + \frac{1}{8}\cos(4x) \end{aligned}$$

3. Use intevals to describe the solution to the inequality $\tan(x) \geq 0$.

Solution: By the graph of $y = \tan(x)$, the solution is

$$x \in \cdots \cup [-\pi, -\tfrac{\pi}{2}) \cup [0, \tfrac{\pi}{2}) \cup [\pi, \tfrac{3\pi}{2}) \cup \cdots$$

In the intervals, left endpoints are included because

$$\cdots = \tan(-\pi) = \tan(0) = \tan(\pi) = \cdots = 0.$$

Right endpoints are not included because $\tan(x)$ is undefined at these points. (This is because $\cos(x) = 0$ at these points and we cannot divide 0.)