

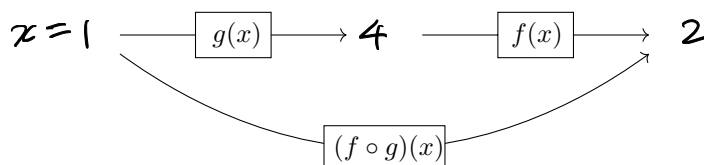
## MATH 2A/5A Prep: Composition of Functions

### Facts to Know:

$f(x)$  and  $g(x)$  are functions, then the composition  $f \circ g$  is defined by:

$$(f \circ g)(x) = f(g(x))$$

Diagram explanation: for example, take  $f(x) = \sqrt{x}$ ,  $g(x) = 3x + 1$ ,



### Examples:

1. Let  $f(x) = e^x$ ,  $g(x) = x^2 - 1$ . Find the functions  $\overbrace{f(g(x))}^{(f \circ g)(x)}$  and  $\overbrace{g(f(x))}^{(g \circ f)(x)}$ , then find  $f(g(1))$  and  $g(f(1))$ .

$$g(x) = u, \quad f(g(x)) = f(u) = e^u = \overbrace{e^{g(x)}}^{(f \circ g)(x)} = e^{x^2 - 1}$$

$$u = f(x) = e^x, \quad g(f(x)) = g(u) = u^2 - 1 = (e^x)^2 - 1 = e^{2x} - 1$$

$$f(g(1)) = e^{1^2 - 1} = e^0 = 1$$

$$g(f(1)) = e^{2 \cdot 1} - 1 = e^2 - 1$$

2. Let  $F(x) = \ln(e^x + 1)$ . Write  $F(x)$  in terms of the elementary functions  $e^x$ ,  $\ln(x)$  and  $mx + b$ , and function composition.

$$x \xrightarrow{h} e^x \xrightarrow{g} e^x + 1 \xrightarrow{f} \ln(e^x + 1)$$

$$F(x) = f(g(h(x))) = (f \circ g \circ h)(x)$$

So if  $h(x) = e^x$ ,  $g(u) = u + 1$ ,  $f(x) = \ln(x)$ ,

3. Let  $f(x) = 2x^2 + 3x + 1$ , simplify the expression  $\frac{f(x+2) - f(x)}{2}$ .