1. Show the trig identity \( 1 + \cot^2(x) = \csc^2(x) \). (See Example 1)

**Solution:** Start with the trig identity \( \sin^2(x) + \cos^2(x) = 1 \), then divide by \( \sin^2(x) \). This gets

\[
\frac{\sin^2(x)}{\sin^2(x)} + \frac{\cos^2(x)}{\sin^2(x)} = \frac{1}{\sin^2(x)}
\]

which simplifies to \( 1 + \cot^2(x) = \csc^2(x) \).

2. Compute \( \cos(3\pi/4) \) and \( \sin(3\pi/4) \).

**Solution:**

The picture on the left illustrates the location of this point on the unit circle corresponding to the angle \( 3\pi/4 \). Notice the triangle that forms has an angle of \( \pi/4 \) from the negative \( x \)-axis. The right image shows that this triangle has sides of length \( \sqrt{2}/2 \), which is both \( \cos(\pi/4) \) and \( \sin(\pi/4) \). Since the point lands in the second quadrant we know the cosine value is negative and the sine value is positive. Therefore

\[
\cos(3\pi/4) = -\frac{\sqrt{2}}{2} \quad \sin(3\pi/4) = \frac{\sqrt{2}}{2}
\]
3. Find $\cos(-\pi/3)$ and $\sin(-\pi/3)$.

Solution:

Since we have a negative angle we go clockwise (not counter-clockwise) around the unit $\pi/3$ radians and thus end in the fourth quadrant. Again in the left diagram we see the related point on the unit circle and the triangle that is formed by adding a line to the positive $x$-axis. On the right is a larger image of this triangle - we get an angle of $\pi/3$ which gives one side of length $1/2$ and one of $\sqrt{3}/2$, corresponding to $\cos(\pi/3)$ and $\sin(\pi/3)$ respectively. Since the point is in the fourth quadrant this means the cosine value is positive and the sine value is negative. Hence

$$\cos(-\pi/3) = \frac{1}{2} \quad \sin(-\pi/3) = -\frac{\sqrt{3}}{2}$$

4. Write $\cos^2(x)$ in terms of $\cos(2x)$.

Solution: Recall the double angle formula $\cos(2x) = 2\cos^2(x) - 1$. Adding 1 to both sides gives $2\cos^2(x) = \cos(2x) + 1$. Dividing by 2 gives

$$\cos^2(x) = \frac{1}{2}(\cos(2x) + 1)$$