Facts to Know:
For simplicity, we assume all functions in this script are continuous and differentiable everywhere.

1. Critical numbers: \( x = c \) is a critical number of the function \( f(x) \) if \( f'(c) = 0 \).

2. First derivative test: Suppose \( c \) is critical number of \( f(x) \)
   - If \( f' \) changes from positive(+) to negative(−), then \( f \) has a local maximum at \( c \), \( x=0 \).
   - If \( f' \) changes from − to +, then \( f \) has a local minimum at \( c \), \( x=0 \).
   - If \( f' \) is + to the left and right of \( c \), or − to the left and right of \( c \), then \( f \) has no local extreme value at \( c \).

3. Second derivative test: If \( f'(c) = 0 \) and
   - \( f''(c) > 0 \), then \( f \) has a local minimum at \( c \), \( x=0 \).
   - \( f''(c) < 0 \), then \( f \) has a local maximum at \( c \), \( x=0 \).
   - \( f''(c) = 0 \), then no conclusion

Example:

1. Find all local extreme values of \( f(x) = x^3 - 3x. \)
   - Find critical numbers:
     \[
     f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1
     \]
     2 critic. numbers: 1 and −1
   - Method 1: First derivative test:
     \[
     \begin{array}{|c|c|c|}
     \hline
     \text{Interval} & f'(x) & f(x) \\
     \hline
     x < -1 & + & \uparrow \\
     -1 < x < 1 & - & \downarrow \\
     x > 1 & + & \uparrow \\
     \hline
     \end{array}
     \]
     \( f(x) \) has:
     local max. \( f(-1)=2 \) at \( x=-1 \)
     local min \( f(0)=-2 \) at \( x=1 \)
   - Method 2: Second derivative test:
     \[
     f''(x) = 6x
     \]
     \[
     \begin{array}{|c|c|c|}
     \hline
     x & f''(x) & f(x) \\
     \hline
     1 & 6 (+) & \text{local min} \\
     -1 & -6 (-) & \text{local max} \\
     \hline
     \end{array}
     \]
     \( f \) has:
     local min \( f(1)=-2 \) at \( x=1 \) and
     local max value \( f(-1)=2 \) at \( x=-1 \)