1. Evaluate the double integral \( \int_D 6(x + y)^2 \, dS \), where \( D \) is the region between the graphs of \( y = x \) and \( y = x^2 \).

**Solution:**

\[
\begin{align*}
\int_D 6(x + y)^2 \, dS &= \int_0^1 \int_{\sqrt{x}}^x 6(x + y)^2 \, dy \, dx \\
&= \int_0^1 \left[ 6x^2 + \frac{6x^3}{3} \right]_{\sqrt{x}}^x \, dx \\
&= \int_0^1 \left[ 6x^2 + 2x^3 - 6\sqrt{x}^2 - 2(\sqrt{x})^3 \right] \, dx \\
&= \int_0^1 \left[ 6x^2 + 2x^3 - 6x - 2x^{3/2} \right] \, dx \\
&= \left[ 2x^3 + \frac{x^4}{2} - 3x^2 - \frac{4x^{5/2}}{5} \right]_0^1 \\
&= 2 - 1 - \frac{6}{5} + \frac{7}{2} \\
&= \frac{71}{70}.
\end{align*}
\]

2. Evaluate the double integral \( \int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1} \, dy \, dx \) by reversing the order of integration.

**Solution:** Let \( D \) be the region described by

\[
D = \{(x, y) \mid 0 \leq x \leq 1, \sqrt{x} \leq y \leq 1\}.
\]

A figure for \( D \) is provided at the end of this solution. By the figure, \( D \) can also be described as

\[
D = \{(x, y) \mid 0 \leq y \leq 1, 0 \leq x \leq y^2\}.
\]
Therefore,

\[
\int_0^1 \int_0^1 \sqrt{y^3 + 1} \, dy \, dx = \iint_D \sqrt{y^3 + 1} \, dS
\]

\[
= \int_0^1 \int_0^y \sqrt{y^3 + 1} \, dx \, dy
\]

\[
= \int_0^1 \left[ x \sqrt{y^3 + 1} \right]_0^y \, dy
\]

\[
= \int_0^1 y^2 \sqrt{y^3 + 1} \, dy
\]

\[
= \int_0^1 \frac{1}{3} \sqrt{u + 1} \, du \quad \text{(where } u = y^3)\]

\[
= \left[ \frac{2}{9} (u + 1)^{\frac{3}{2}} \right]_0^1
\]

\[
= \frac{4\sqrt{2} - 2}{9}
\]

Figure for \(D\):

\[
y = \sqrt{x}
\]