MATH 2E Prep: Partial derivatives

Facts to Know:

1. Rules for finding partial derivatives of \( u = f(x, y, z) \)
   - Notation: \( f_x(x, y, z) = \frac{2u}{\partial x} = \frac{df}{\partial x} = \frac{\partial}{\partial x} f(x, y, z) \)
   - To find \( f_x \), regard \( y, z \) as constants and differentiate \( f(x, y, z) \) with respect to \( x \).
   - To find \( f_y \), regard \( x, z \) as constants and differentiate \( f(x, y, z) \) with respect to \( y \).
   - Similar for \( f_z \).

2. Definition of gradient:
   - \( f(x, y, z) \) function of 3 variables, \( \nabla f = \left< \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right> \)

Examples:

1. Calculate the gradient of the function \( f(x, y, z) = x + ye^z \), and its value at two points \((1, 0, 2)\) and \((1, 1, 0)\).

\[
\frac{\partial f}{\partial x} = 1, \quad \frac{\partial f}{\partial y} = 0 + 1e^z, \quad \frac{\partial f}{\partial z} = 0 + ye^z
\]

\[
\Rightarrow \nabla f = \left< \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right> = \left< 1, e^z, ye^z \right>
\]

\[
(\nabla f)(1, 0, 2) = \left< 1, e^2, 0 \cdot e^2 \right> = \left< 1, e^2, 0 \right>
\]

\[
(\nabla f)(1, 1, 0) = \left< 1, e^0, 1e^0 \right> = \left< 1, 1, 1 \right>
\]