MATH 3D Prep: Improper Integration

Facts to Know:

For simplicity, assume all functions in this section are defined , continuous and differentiable on $[0, \infty)$.

1. Definition:
   - $\int_0^\infty f(x)dx = \lim_{b \to \infty} \int_0^b f(x)dx$
   - Let $F(x)$ be antiderivative of $f(x)$, then $\int_0^\infty f(x)dx$ converges if $\lim_{x \to \infty} F(x)$ exist and finite
   - If converges, $\int_0^\infty f(x)dx = \lim_{b \to \infty} \left[F(b) - F(0)\right] = \lim_{b \to \infty} F(b) - F(0)$
2. l-Hoptal's Rule:
   - Important ! Need to check.
   - Computes $\lim_{x \to \infty} \frac{f(x)}{g(x)}$, when the limit is $\frac{0}{0}$-type or $\frac{\infty}{\infty}$-type.
   - $\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}$

Examples:

1. Evaluate $\int_0^\infty e^{-3t}dt = \lim_{b \to \infty} \int_0^b e^{-3t}dt = \lim_{b \to \infty} \left[-\frac{1}{3}e^{-3t}\right]_0^b$

   $= \lim_{b \to \infty} \left[-\frac{1}{3}e^{-3b} + \frac{1}{3}e^0\right] = \lim_{b \to \infty} \left[\underline{-\frac{1}{3}\cancel{e^{-3b}}} + \frac{1}{3}\right]$ 0

   $= 0 + \frac{1}{3} = \frac{1}{3}$

2. Evaluate $\int_0^\infty te^{-t}dt$

   $= \lim_{b \to \infty}$