

## MATH 3D Prep: Integrals Involving Extra Variable

### Facts to Know:

There are two ways to involve an extra variable in integrals:

1. Fundamental Theorem of Calculus Part I:  $g(x) = \int_a^x f(t) dt$  satisfies  $g'(x) = f(x)$ .

2. Extra variable in  $f(x)$ :  $\int_a^b f(x, t) dt$  is a function of  $x$ .

*when compute, regard  $x$  as constant.*

Both of these two types of integrals are used in Math 3D.

### Examples:

1. Find a function  $g(x)$  such that  $g'(x) = e^{x^2}$  and  $g(2) = e^4$  *want to choose  $a=2$*

$$g(x) = \int e^{x^2} dx = ??? \quad \text{not work.}$$

Solution: write  $g(x) = \int_a^x e^{t^2} dt$ ,  $a=2$ , get

$$g(x) = \int_2^x e^{t^2} dt + C$$

$$g(2) = e^4, \text{ get } e^4 = g(2) = \int_2^2 e^{t^2} dt + C \Rightarrow C = e^4 \Rightarrow g(x) = \int_2^x e^{t^2} dt + e^4$$

2. Evaluate  $\int_0^1 t e^{-xt} dt$ . ( $x \neq 0$ )

$$u=t, \quad dv=e^{-xt} dt$$
$$du=dt, \quad v=-\frac{1}{x} e^{-xt}$$
$$\int_0^1 t e^{-xt} dt = \int_0^1 u dv = \left. -\frac{t}{x} e^{-xt} \right|_0^1 -$$