MATH 3D Prep: Linear Independence

Facts to Know:

- A set of vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ is *linearly independent* if:

  whenever $a_1\vec{v}_1 + a_2\vec{v}_2 + \cdots + a_n\vec{v}_n = \vec{0}$,

  we have $a_1 = a_2 = \cdots = a_n = 0$.

- Polynomials of degree ≤ 2 can be viewed as vectors in $\mathbb{R}^3$:

  $$ax^2 + bx + c \longleftrightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Examples:

1. Determine whether the vectors $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$ are linearly independent.

$a_1\vec{u} + a_2\vec{v} + a_3\vec{w} = \vec{0}$, L.I. $\iff$ system has only solution $a_1 = a_2 = a_3 = 0$.

Aug. matrix: $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

no pivot, free var.

$\Rightarrow$ a_3 - free.

$\Rightarrow$ have nonzero solution $\Rightarrow$ linearly dependent.

2. Determine whether the polynomials x, $x+1$, and x^2 are linearly independent.

Method 1: $x = 0x^2 + 1 \cdot x + 0 \rightsquigarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$x+1 = 0x^2 + 1 \cdot$