MATH 3D Prep: Eigenvalue and Eigenvectors

Facts to Know:

A is an $n \times n$ matrix:

- λ $\in \mathbb{R}$ is an *eigenvalue* of A if there is a nonzero vector $\vec{v}$ ($n \times 1$ vector) such that $A\vec{v} = \lambda\vec{v}$.
- Such $\vec{v}$ is called an eigenvector of A corresponding to eigenvalue λ.
- λ is an eigenvalue if and only if $\det(A - \lambda I) = 0$.
- The set of all solutions to $(A - \lambda I)\vec{x} = \vec{0}$ is called the *eigenspace* of A corresponding to λ.

Examples:

1. Let $A = \begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix}$, find all eigenvalues, and for each eigenvalue, find a basis for the corresponding eigenspace.

$$0 = \det(A - \lambda I) = \begin{vmatrix} 5-\lambda & 2 \\ 1 & 4-\lambda \end{vmatrix} = (5-\lambda)(4-\lambda) - 2 = \lambda^2 - 9\lambda + 18$$
$$= (\lambda - 3)(\lambda - 6)$$

$\Rightarrow \lambda = 3$ or $\lambda = 6$, 2 eigenvalues.

For $\lambda = 3$, solve for $(A - \lambda I)\vec{x} = \vec{0}$, $\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}\vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

augmented matrix: $\left(\begin{array}{cc|c} 2 & 2 & 0 \\ 1 & 1 & 0 \end{array}\right) \underset{R_1 \to R_1/2}{\sim} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right) \underset{R_2 \to R_2 - R_1}{\sim} \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

pivot

free var.

$\Rightarrow$?