MATH 3D Prep: Sigma Notations

Facts to Know:

A way to elegantly write sums:

- $1 + 4 + 9 + \cdots + n^2 = \sum_{i=1}^{n} i^2 = (1+0)^2 + (1+1)^2 + (1+2)^2 + \cdots + (1+n-1)^2 = \sum_{j=0}^{n-1} (1+j)^2$ ($c = -1$, $j = i-1 \Rightarrow i = j+1$)
- $a_3 + a_4 + a_5 + \cdots + a_{20} = \sum_{i=3}^{20} a_i$
- $b_0 + b_1 + b_2 + \cdots = \sum_{i=0}^{\infty} b_i = \sum_{n=0}^{\infty} b_n$

Shifting the index:

- Writing the sum in sigma notation in another way.
- Shifting the index up by c means replacing the index i by $j =$ $i + c$ everywhere.

Examples:

1. Rewrite the sum $\sum_{i=1}^{n} \frac{\cos(i-1)}{i}$ as a sum that starts from $i = 0$.

replace i by $j = i + (-1) = i - 1$, then $i = j + 1$

$$\sum_{i=1}^{n} \frac{\cos(i-1)}{i} = \sum_{j+1=1}^{j+1=n} \frac{\cos(j+1-1)}{j-1} = \sum_{j=0}^{j=n-1} \frac{\cos(j)}{j-1} = \sum_{i=0}^{n-1} \frac{\cos(i)}{i-1}$$

2. Write the power series $x \sum_{n=0}^{\infty} \frac{x^n}{n!}$ in the form $\sum_{n=c}^{\infty} a_n x^n$.

$$x \sum_{n=0}^{\infty} \frac{x^n}{n!} = -$$