Let $f(x,y) = xy^2$.

1. What is the gradient of $f$ at the point $(0,1)$?

**Solution:** We have that $f_x = y^2$ and $f_y = 2xy$. Thus, $\nabla f(0,1) = (1,0)$.

2. What direction increases the fastest from the point $(0,1)$ and what is the rate of change in the same direction?

**Solution:** The direction that increases the fastest from the point $(0,1)$ is the vector $(1,0)$ with rate of change $||\nabla f(0,1)|| = 1$.

3. What is the linearization of $f$ at the point $(0,1)$?

**Solution:** The linearization is

$$L_f(x,y) = f(0,1) + \nabla f(0,1) \cdot ((x,y) - (0,1))$$

$$= 0 + (1,0) \cdot ((x,y) - (0,1))$$

$$= x.$$