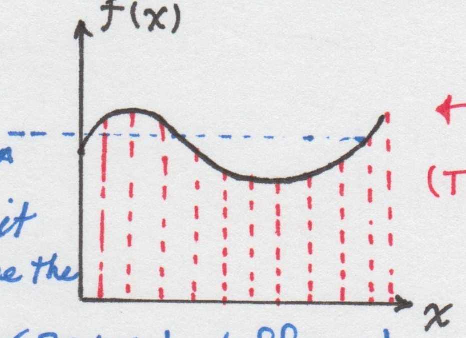
# Four ways to Represent a function

## Mathematical Definition of a function:

A function $f$ is a rule that assigns to each element $x$ in set D, exactly one element called $f(x)$ in set E.

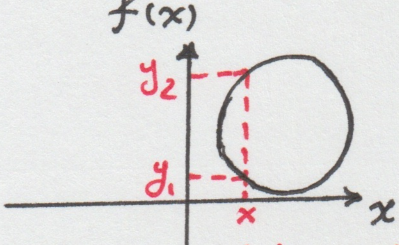
for each $x$ there is only one $f(x)$

Example:




(This is a function)

Remember, it is ok to have the same $f(x)$ for several different values of $x$, but we can NOT have several different $f(x)$ for the same $x$.




This is NOT a function

$\{(2,3),(2,4),(4,5)\}$
Not a function

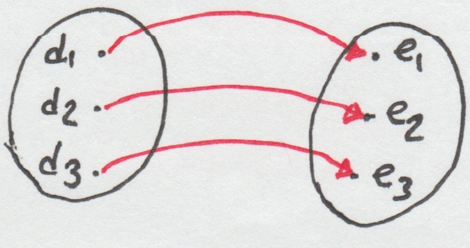
$\{(2,3),(4,3),(5,6)\}$
This is a function

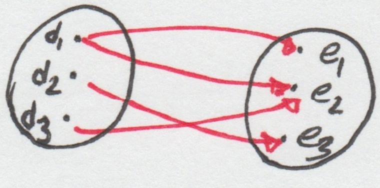
In general we show functions as $f(x)$, $f(t)$, $h(s)$, ... or anything else and we read it as "f of x"

$x$: is the independent variable

$f(x)$ or $y$: is the dependent variable

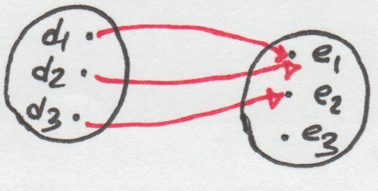
You can also have arrow diagram:




It is NOT a Function

why? Think about it



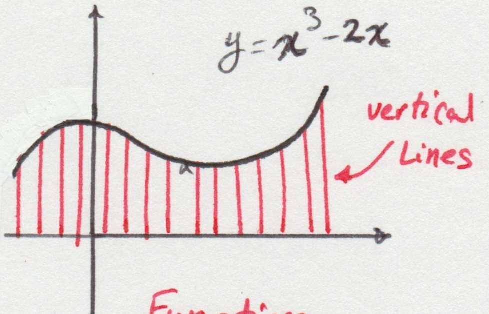

It is a function

why? Think about it.

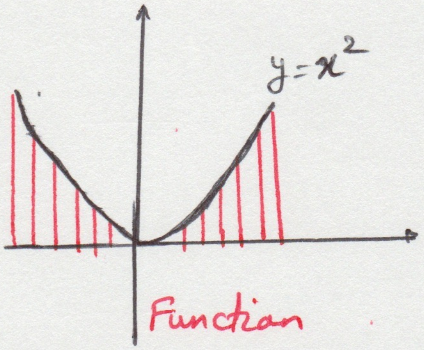
# Four ways to represent a function

If we have a curve in $xy$ plane, if and only if no vertical line intersects with the curve more than once, then the curve is the graph of a function of $x$

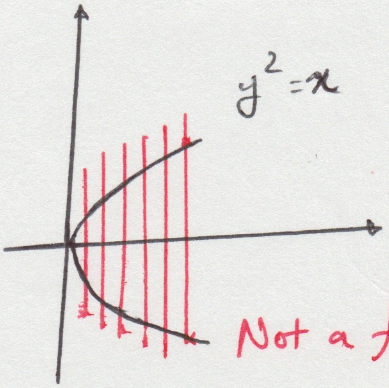
This is called "The Vertical Line Test"




Function




Function




Not a function

Functions arise whenever one quantity depends on another.

Example: The Area of a circle depends on its radius.

$$\Rightarrow A(r) = \pi r^2$$

The cost of mailing an envelope depends on its weight.

Cost $(c)$
weigh $(w)$ $\Rightarrow$ $C(w)$

Now that we have talked about functions, lets see how we can represent them (in other words, different ways to represent a function)

Answer:

1) Verbally $\longrightarrow$ (By description in words)
2) Numerically $\longrightarrow$ (By a Table of values)
3) visually (Graphically) $\longrightarrow$ (By a graph)
4) Algebraically $\longrightarrow$ (By an explicit formula)

# Four Ways to represent a function

**Examples:** 1) Verbally: The area of a circle is pi times the square of radius

The cost of a train ticket depends on the distance from the station to the destination.

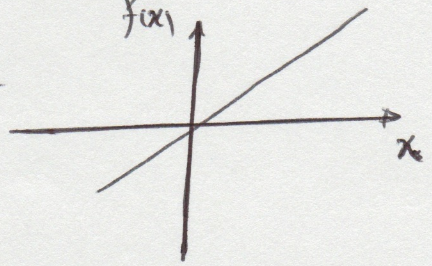
2) Numerically

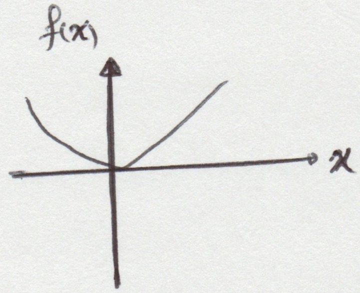
| time | population of bacteria in a test tube |
|---|---|
| 0 | 100 |
| 2 hours | 200 |
| 4 hours | 400 |
| 6 hours | 800 |
| 8 hours | 1600 |

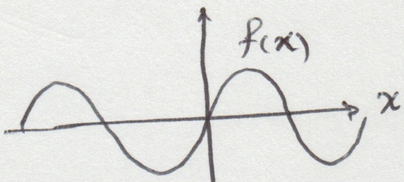
$\{(2,3), (3,5), (6,8)\}$

$\{(0,100), (2,200), (4,400), (6,800), (8,1600)\}$

3) visually




$f(x) = \sin(x)$

$f(x) = x^2$

4) Algebraically $f(x) = x$

## Lets see what is the Domain & Range of a function:

You remember the def. of function:

A function $f$ is a rule that assigns to each element $x$ in a set $D$, exactly one element called $f(x)$, in a set $E$

All $x$s $\in D \rightsquigarrow$ Domain (All posible $x$'s create domain)

All $f(x) \in E \rightsquigarrow$ Range (All possible $f(x)$'s create Range)

**Example:** Find the domain of these two functions:

$f(x) = \sqrt{x+2}$

$g(x) = \dfrac{1}{x^2 - x}$

what are the acceptable $x$s

$x^2 - x$ should Not be zero

$x \neq 0$, $x \neq 1$

$x + 2 \geq 0$

$x \geq -2 \longrightarrow [-2, \infty)$

$D = \{x \mid x \neq 0, x \neq 1\}$

**Difference Quotient**: Occurs frequently in Calculus (it represent the average rate of changes of a function like $f(x)$ between $x=a$ & $x=a+h$  D.Q $= \dfrac{f(a+h)-f(a)}{h}$

**Example:** find the difference Quotient of function $f(x) = x^2 - 3$ between $x=3$ and $x=3+h$

$x=3 \to f(x) = x^2 - 3 = 6$

$x=3+h \to f(3+h) = (3+h)^2 - 3 = 9 + h^2 + 6h - 3 = h^2 + 6h + 6$

$$\frac{f(3+h)-f(3)}{h} = \frac{h^2+6h+\cancel{6}-\cancel{6}}{h} = h+6$$

**Def. of Piecewise Defined Functions:**

They are functions that are defined by different formulas in different parts of their domain.

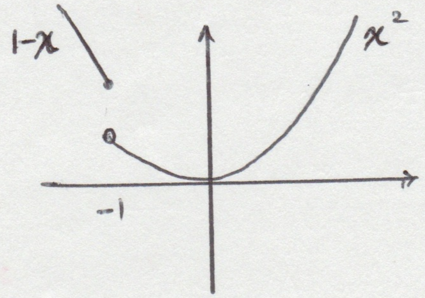
**Example:** $f(x) = \begin{cases} 1-x & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$

Evaluate $f(-2)$, $f(-1)$ & $f(0)$

$f(0) = (0)^2 = 0$

$f(-1) = 1-(-1) = 2$

$f(-2) = 1-(-2) = 3$



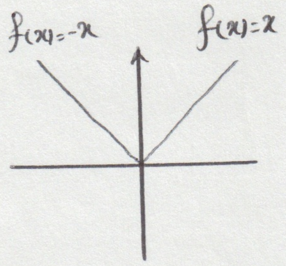

**Example:** $f(x) = |x|$

what does absolute value do?

$|-3| = 3 \qquad |3| = 3$

$|-2| = 2 \qquad |2| = 2$

$f(x) = |x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$



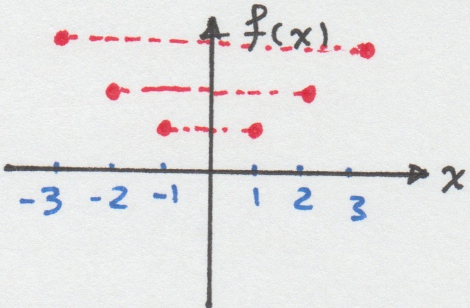
## Odd and Even Functions:

If a function $f$ satisfies $f(-x) = f(x)$ for every $x$ in the domain, Then $f$ is called an even function.

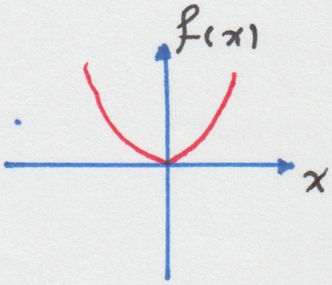
$f(x) = x^2 \longrightarrow f(-x) = (-x)^2 = x^2$ Therefore $f$ is an even function

What does this mean graphically?

Means for every value of $x$ That we have a $f(x)$ value, we have The same $f(x)$ for $-x$.




Result: $f$ is symmetric with respect to $y$ axis.




If a function $f$ satisfies $f(-x) = -f(x)$ for every $x$ in The domain, Then $f$ is called an odd function.

$f(x) = x^3 \longrightarrow f(-x) = -f(x)$? $\quad f(-x) = (-x)^3 = -x^3 = -f(x)$ ✓

$f(x) = \sin(x) \rightarrow f(-x) = \sin(-x) = -\sin(x) = -f(x)$

Result: The graph of an odd function is symmetric about the Origin.