

Mathematical Models & Different Types of Functions:

A Mathematical model is a mathematical description (often by means of functions) or (equations) of a real-world phenomenon.

such as: speed of a falling object

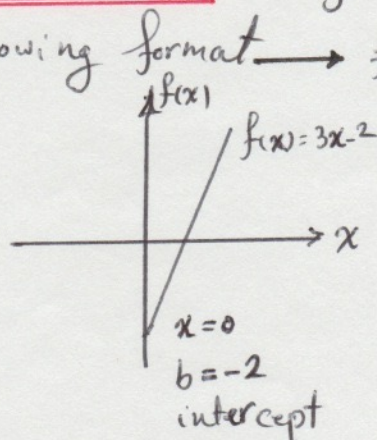
population

concentration of a product in a chemical reaction

life expectancy of a person @ birth and ...

There are many different types of functions that can be used to model relationships observed in real world.

Linear Models: The graph of the function is a line and it follows the following format $\rightarrow f(x) = mx + b$



slope

intercept

↳ where the line intersects y axis.

$m=3$
slope

polynomial function: a function f is called polynomial if:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

$a_0, a_1, \dots, a_n$ are called polynomial coefficients

if $a_n \neq 0$, n is the degree of polynomial

↳ Leading Coefficient

Example: $f(x) = 2x^6 - x^4 + \frac{2}{5}x^3 + \sqrt{2}$ the polynomial is of degree 6

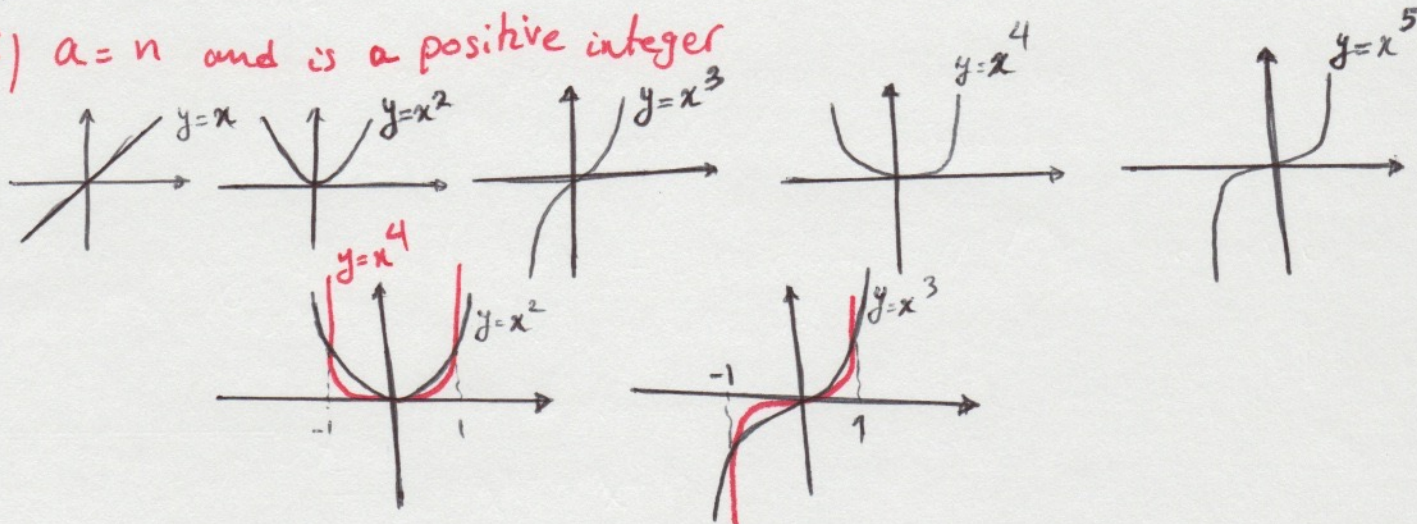
there are two famous types of polynomials

$$\begin{cases} f(x) = ax^2 + bx + c & \text{quadratic} \\ f(x) = ax^3 + bx^2 + cx + d & \text{cubic} \end{cases}$$

3- Power function: A function of the form $f(x) = x^a$, where a is a constant is called a power function.

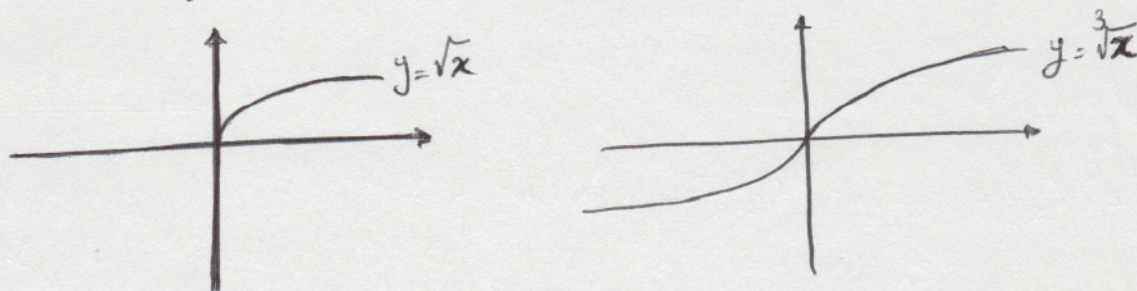
Depending on the value of a we can have different shapes for power functions.

i) $a = n$ and is a positive integer

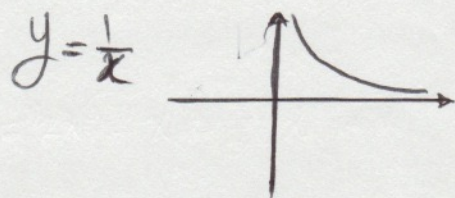


ii) $a = \frac{1}{n}$ where n is a positive integer

$y = \sqrt{x}$ $y = \sqrt[3]{x}$ root 3 or Cube root
Square root



iii) $a = -1$ (Also called reciprocal function)



rational functions: f is the ratio of two polynomials

$$f(x) = \frac{P(x)}{Q(x)}$$

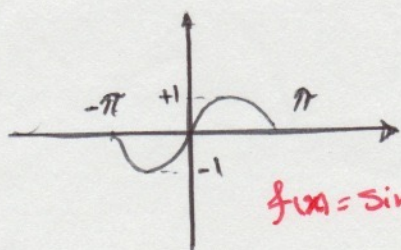
Example: $f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4} \rightarrow \text{Domain} = \{x \mid x \neq 2, x \neq -2\}$

Algebraic functions: f is an algebraic function if it can be constructed using algebraic operations (such as multiplication, division, addition, subtraction, taking roots). They can assume variety of shapes.

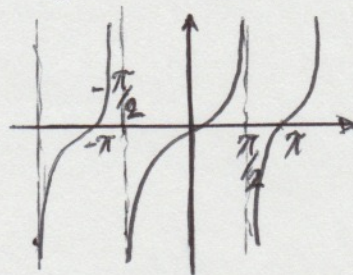
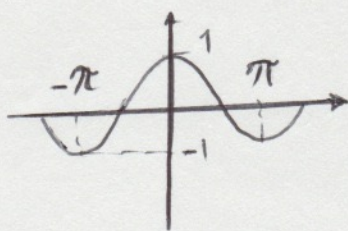
Trigonometric Function:

They include sine, cosine, and tangent $\rightarrow \begin{cases} \sin \\ \cos \\ \tan \end{cases}$

$f(x) = 2 \sin^2 x + \cos^2 x$ is a trigonometric function.

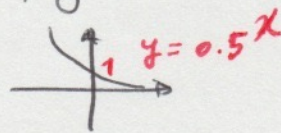
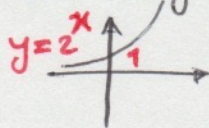


$f(x) = \sin(x)$



Exponential functions: the exponential functions have the form

$f(x) = a^x$, we will study them extensively later, just know that whether $a > 1$ or $a < 1$



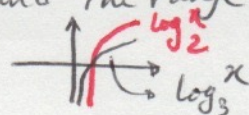
Logarithmic functions:

$$f(x) = \log_a x$$

$a \rightarrow$ base (a positive constant)

it is the inverse of exponential func.

for a logarithmic function the domain is $(0, \infty)$ and the range is $(-\infty, \infty)$



Example: Classify the following Functions:

a) $f(x) = 5^x$ exponential Function

c) $g(x) = x^5$ Power Function

b) $h(x) = \frac{1+x}{1-\sqrt{x}}$

Algebraic function
but Not rational

d) $u(t) = 1 - t + 5t^4$ Polynomial