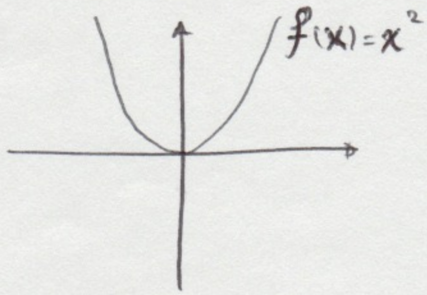
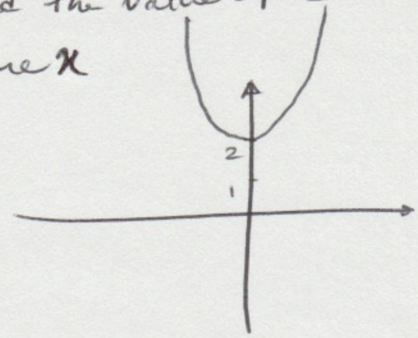
Transformations of functions:

By applying certain transformations to the graph of a given function, we can obtain the graph of certain related functions. This gives us the Ability to sketch the graph of many functions by hand.

Example: sketch the graph of function $f(x) = x^2 + 2$



now whatever value you have for $f(x)$, we just add the value of 2 to that for the same x

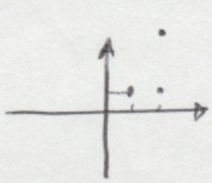


Translation Transformations:

These are the transformations that shift graphs upward, downward, to the left or to the right.

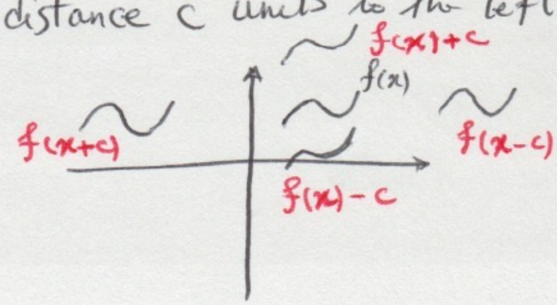
suppose that c is a possitive number $c > 0$ and we have the graph of $f(x)$:

1. $y = f(x) + c$, shifts the graph of $y = f(x)$ a distance c units upward.
2. $y = f(x) - c$, shifts the graph of $y = f(x)$ a distance c units downward.
3. $y = f(x - c)$, shifts the graph of $y = f(x)$ a distance c units to the right
4. $y = f(x + c)$, shifts the graph of $y = f(x)$ a distance c units to the left



$f(1) = 1$ $\quad f(1-1)$

$f(2) = 4$ $\quad f(2-1) = f(1) = 1$



stretching Transformations:

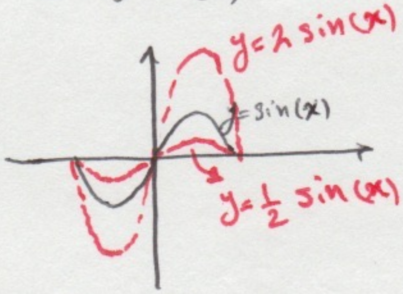
$y = cf(x)$ stretches the graph of $y = f(x)$ vertically by a factor of c

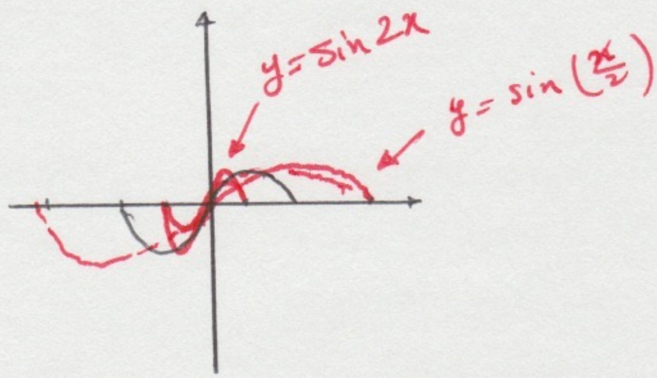
for the same reason

$y = \frac{1}{c} f(x)$ shrinks the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$ shrinks the graph of $y = f(x)$ horizontally by a factor c

$y = f(\frac{x}{c})$ stretches the graph of $y = f(x)$ horizontally by a factor c

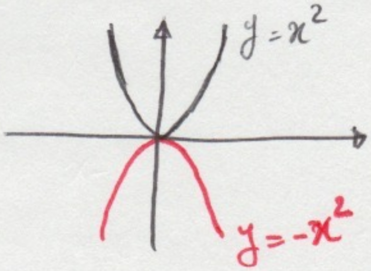


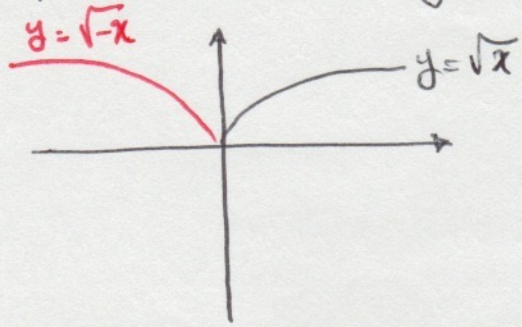


Reflecting Transformations:

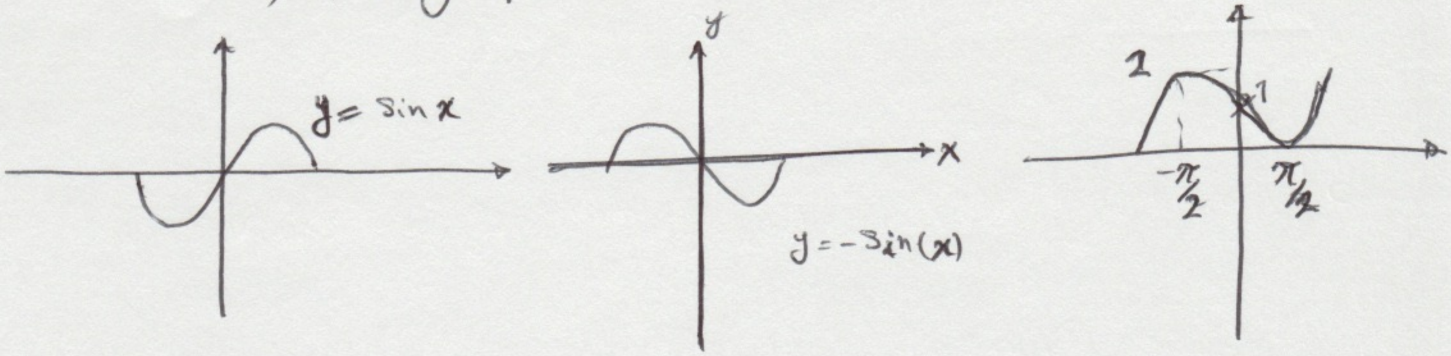
$y = -f(x)$ reflects the graph of $f(x)$ about the x-axis

$y = f(-x)$ reflects the graph of $f(x)$ about the y-axis





Example: plot (sketch) the graph of $f(x) = 1 - \sin(x)$



Composite functions : given Two functions f and g the composite function $f \circ g$ is defined as:

$$f \circ g\,(x) = f(g(x))$$

"f circle g"

What does it mean? suppose that you have a function $f(t) = t^2 + \sin t$, also suppose that t itself is a function of x such that $t(x) = 2x^2 + 1$

$$f \circ t = f(t(x)) = f(2x^2+1) = (2x^2+1)^2 + \sin(2x^2+1)$$

Another Example: if $f(x) = x^2$ and $g(x) = x - 3$ find the composite functions $f \circ g$ and $g \circ f$?

$$f \circ g = f(g(x)) = [g(x)]^3 = (x-3)^3$$

$$g \circ f = g(f(x)) = f(x) - 3 = x^2 - 3$$