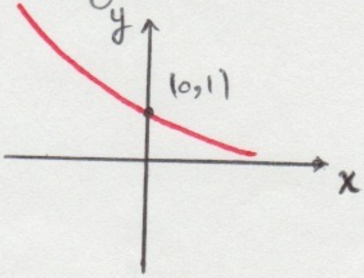
Exponential Functions

We previously defined a function in the form of $f(x) = a^x$ (x being the independent variable). Do <u>NOT</u> confuse exponential functions with power functions ($f(x) = x^a$ and the independent variable appears in the base, Not the exponent)

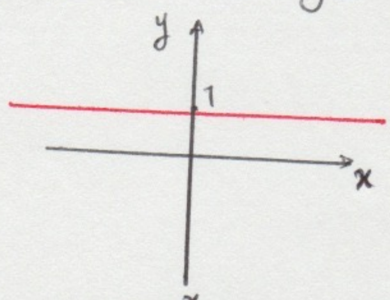
what is the meaning of $f(x) = a^x$ (it means, multiply a by itself x times)

$$f(x) = a^x = \underbrace{a \times a \times a \times \cdots \times a}_{x \text{ factors}}$$

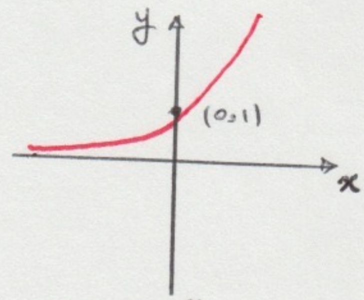
Depending on the value of a we can get these Three forms



a) $y = a^x$, $0 < a < 1$



b) $y = 1^x$



c) $y = a^x$, $a > 1$

Laws of Exponents:

1) $a^{x+y} = a^x \cdot a^y$

$a^x = \underbrace{a \times a \times \cdots a}_{x \text{ times}}$

$a^y = \underbrace{a \times a \times a \cdots \times a}_{y \text{ times}}$

$a^x \cdot a^y = \underbrace{(a \times a \times \cdots a)}_{x \text{ times}} \underbrace{(a \times a \times \cdots a)}_{y \text{ times}}$

Means that a is being multiplied by itself $x+y$ times

2) $a^{x-y} = \dfrac{a^x}{a^y}$

Following the similar logic,

$$\frac{a^x}{a^y} = \frac{\overbrace{(a \times a \times \cdots a)}^{x \text{ times}}}{\underbrace{(a \times a \times \cdots a)}_{y \text{ times}}}$$

the numerator and the denominator cancel out each other

$$\Rightarrow \frac{a^x}{a^y} = \frac{\overbrace{a \times a}^{(x-y \text{ times})} \times \cancel{\cdots} \times \cancel{a}}{(\cancel{a} \times \cancel{a} \times \cdots \cancel{a})}$$

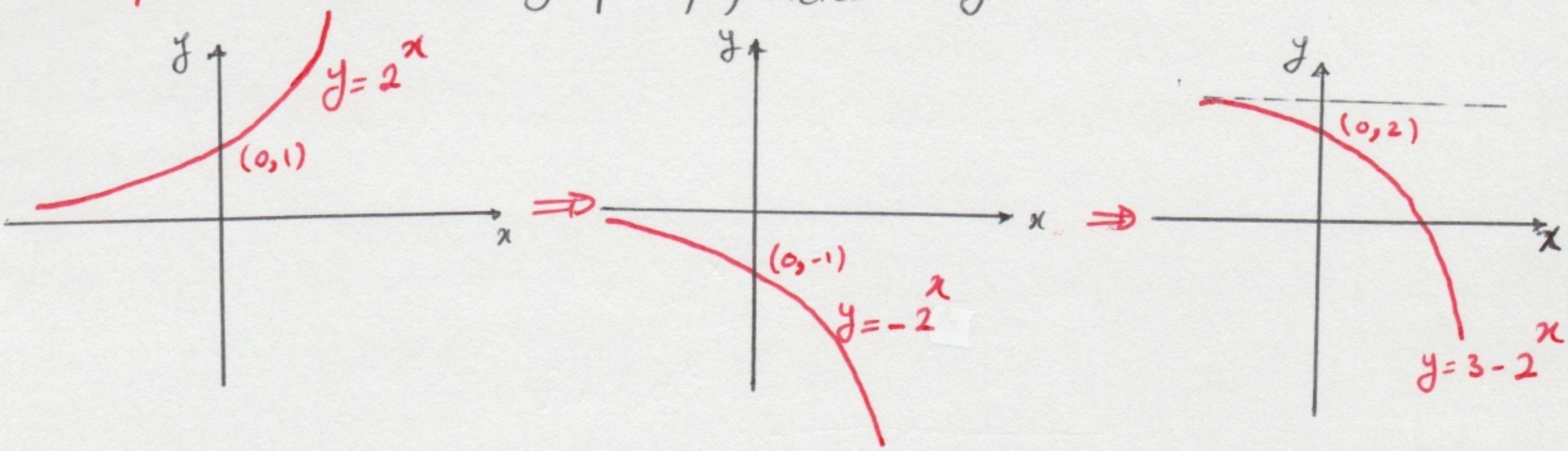
3) $(a^x)^y = a^{xy}$

4) $(ab)^x = a^x b^x$

Prove These last two as an exercise.

Exponential Functions

Example: sketch the graph of function $y = 3 - 2^x$.



Applications of Exponential Functions:

Exponential Functions occur very frequently in mathematical models of nature & society.

e.g. Population Growth of Bacteria (The # of bacteria at time t is $p(t)$, in other words, the population of bacteria is a function of time.

Suppose the initial population is 1000 ($P(t=0)$ or $P(0) = 1000$)

After 1 hour the population is doubled $\Rightarrow$ $P(1) = 2000$

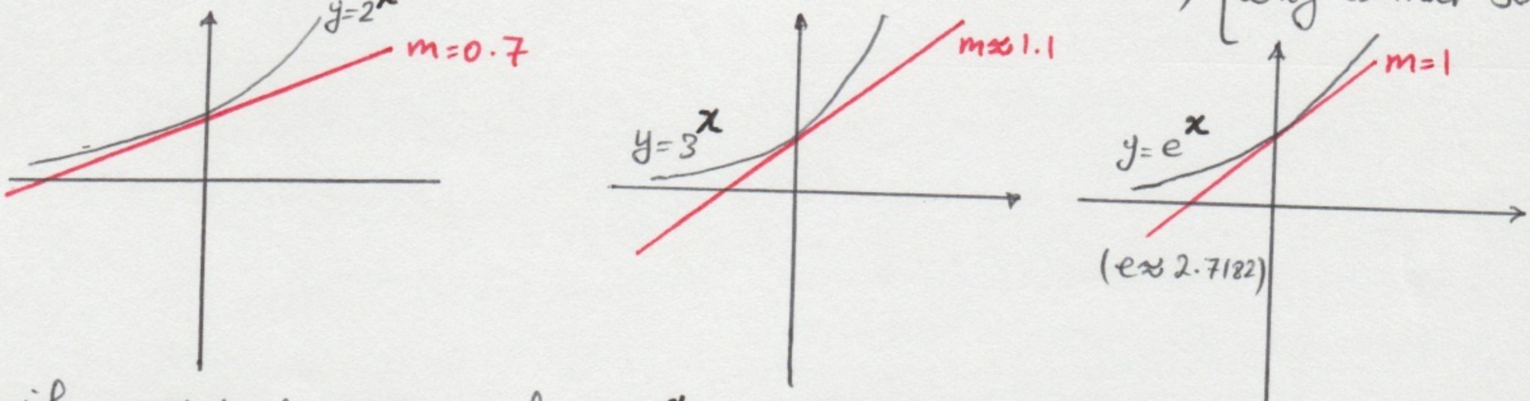
After Another hour the population is again doubled $\Rightarrow$ $P(2) = 4000$

We can write This as the following General Rule

$$P(t) = 2^t \times 1000$$

The number e ($e = 2.71828 \ldots$)

of all possible bases for an exponential function, there is one that is most convenient for the purposes of Calculus) [why is that So?]



if we plot the graph of $y=2^x$, $y=3^x$, $y=e^x$ → All these Curves pass Through point $(0,1)$ but the slope of The tangent line is different. for example for $y=2^x$ the slope at point $(0,1)$ is about 0.7, for $y=3^x$ it is about 1.1 but there is only one base that its tangent is $m=1$ and That base is about 2.7182 in which from this point on we will Call it $\boxed{e}$