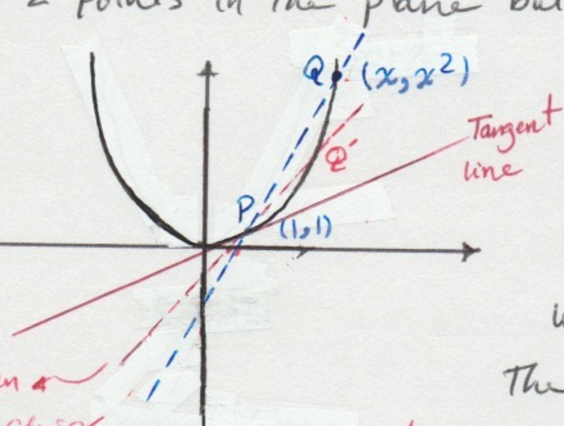
Tangent line probem & Velocity problem

section 2.1

problem statement: Suppose that we have a curve like $y=x^2$ and we want to know the slope of the tangent line at point $P(1,1)$.

The problem is that in order to calculate The slope of a line, we need to have at least 2 points in The plane but right now we only have one (Point $P(1,1)$)



This happens when we pick point Q closer to point P. The slope of PQ' line is closer to the Tangent line

Since there is no way to find The slope of The tangent line here we use a trick.

We say, I find another point on The graph of $f(x)=x^2$ and call This point Q. Then I find a line that passes through points P & Q and if Q is close to P (very close) I can approximate the new PQ line as The tangent line.

Now the slope of $m_{PQ} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$P(1,1)$

$Q(x, x^2) \Rightarrow m_{PQ} = \dfrac{x^2-1}{x-1}$

if I pick diff values for Q (its x value) The slope will be as the following

x	m_{PQ}
2	3
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001

x	m_{PQ}
0	1
0.5	1.5
0.9	1.9
0.99	1.99
0.999	1.999

There is a pattern Here. The closer the value of x for point Q is to The point P, The closer The value of slope would be to 2/

Tangent line problem — Section 2.1

Velocity Problem

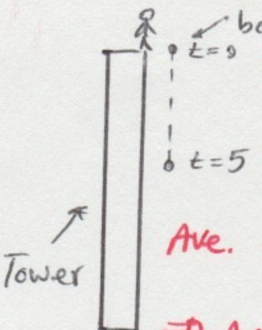
Problem statement: Lets look at another problem. Suppose that we go to a tower balcony which it height is 450m. we are interested in the velocity of the ball exactly at $t=5$. (Instantaneous velocity). We know that the velocity of the ball (avererage velocity) is equal to the distance traveled devided by the length of time). we also know that the distance that the ball travels as a function of time is

$$s(t) = 4.9t^2$$

$$\text{Average Velocity} = \frac{\text{Distance (Displacement)}}{\text{time}}$$

But this problem does NOT ask for average velocity, it asks about instantaneous velocity. (velocity exactly at $t=5$)

The trick to Do This is to try finding average velocity at very short intervals which at very very small intervals can be considered as the instantaneous velocity.



Distance traveled in 5 seconds

$$\text{Ave. velocity} = \frac{\Delta s}{\Delta t} = \frac{S(5) - S(0)}{5 - 0}$$

$$\Rightarrow \text{Ave. velocity} = \ldots$$

Now instead of the interval from $t=0$ to $t=5$ we calculate the average velocity from $t=5$ to $t=6$

$$\Rightarrow \text{Ave. vel.} = \frac{S(6) - S(5)}{6 - 5} = 53.9 \ \text{m/s}$$

We keep making this interval smaller **and** smaller. ⇒

time interval	Ave. velocity (m/s)
$5 < t < 6$	53.9
$5 < t < 5.1$	49.49
$5 < t < 5.05$	49.245
$5 < t < 5.01$	49.049
$5 < t < 5.001$	49.0049

The pattern shows us that as the interval gets smaller and smaller, the average velocity tends to get close to a constant # of 49.00 m/s which is our INSTANTANEOUS velocity