you might think that a real life problem involving linear algebra would only have one solution or perhaps no solution. contrary to this belief, there are a lot of situations that a linear system may have many solutions. In this section, we will investigate some of these situations.

Balancing Chemical Equations:

chemical equations describe the quantities of substances consumed and produced by chemical reactions. for instance, when propane gas burns (C_3H_8), it combines with oxygen O_2 and produces carbon monoxide & water. One way of balancing this reaction is to use vector equations to describe the number of atoms of each type present in a reaction.

Ex. Balance the chemical equation for propane gas burning, using the following unbalanced chemical reaction.

$$C_3H_8 + O_2 \longrightarrow CO_2 + H_2O$$

since we donot know how many of each component is available, we write down the following general eq.

$$(x_1)\, C_3H_8 + (x_2)\, O_2 \longrightarrow (x_3)\, CO_2 + (x_4)\, H_2O$$

we see that only oxygen, hydrogen, and carbon atoms are involved in this reaction.

Therefore, if we think of a vector containing the atoms of C, H, & O for each of these components, we see that

$$C_3H_8 : \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} \qquad O_2 : \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \qquad CO_2 : \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \qquad H_2O : \begin{array}{c} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{Carbon} \\ \leftarrow \text{Hydrogen} \\ \leftarrow \text{Oxygen} \end{array}$$

$$\Rightarrow x_1\begin{bmatrix}3\\8\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\end{bmatrix} = x_3\begin{bmatrix}1\\0\\2\end{bmatrix} + x_4\begin{bmatrix}0\\2\\1\end{bmatrix}$$

$$\begin{bmatrix}3 & 0 & -1 & 0\\8 & 0 & 0 & -2\\0 & 2 & -2 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \xrightarrow[\text{Matrix}]{\text{Augmented}} \begin{bmatrix}3 & 0 & -1 & 0 & 0\\8 & 0 & 0 & -2 & 0\\0 & 2 & -2 & -1 & 0\end{bmatrix}$$

cale & $-8R_1 + R_2$

$$\begin{bmatrix}1 & 0 & -1/3 & 0 & 0\\0 & 0 & 8/3 & -2 & 0\\0 & 2 & -2 & -1 & 0\end{bmatrix} \xrightarrow[\text{The rows}]{\text{switch}} \begin{bmatrix}1 & 0 & -1/3 & 0 & 0\\0 & 2 & -2 & -1 & 0\\0 & 0 & 8/3 & -2 & 0\end{bmatrix}$$

scale

$\frac{R_3}{4} + R_1$

$R_3 + R_2$

$$\begin{bmatrix}1 & 0 & -1/3 & 0 & 0\\0 & 1 & -1 & -1/2 & 0\\0 & 0 & 4/3 & -1 & 0\end{bmatrix} \longrightarrow \begin{bmatrix}1 & 0 & 0 & -1/4 & 0\\0 & 1 & 0 & -5/4 & 0\\0 & 0 & 4/3 & -1 & 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0 & -1/4 & 0\\0 & 1 & 0 & -5/4 & 0\\0 & 0 & 1 & -3/4 & 0\end{bmatrix}$$

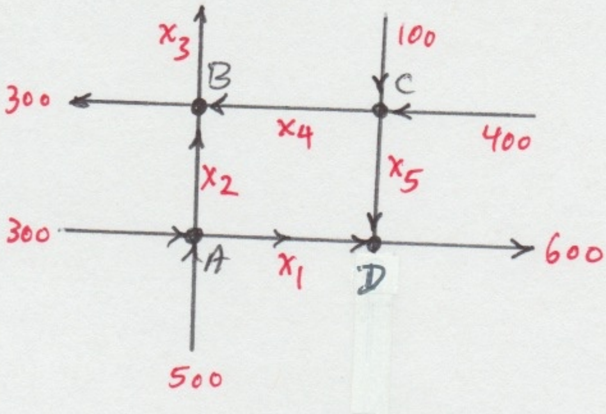
since the coefficients in a chemical reaction must be integers, we take $x_4 = 4$ which assures That we donot get fractions for x_1, x_2, & x_3.

if $x_4 = 4 \longrightarrow x_1 = 1$, $x_2 = 5$, & $x_3 = 3$

$$C_3H_5 + 5\,O_2 \longrightarrow 3\,CO_2 + 4\,H_2O$$

we can see that we could double all the coefficients if we wanted to.

Example: The network in the following figure shows the trafic flow (in vehicles per hour) over several one-way streets during a typical day. Determine the general flow pattern for the network.



we can write the equations that describe the flow and then find the general solution of the system

Intersection	Flow In	Flow out
A	$300+500$	x_1+x_2
B	x_2+x_4	x_3+300
C	$400+100$	x_4+x_5
D	x_1+x_5	600

$$\Rightarrow \begin{cases} x_1+x_2=800 \\ x_2+x_4-x_3=300 \\ x_4+x_5=500 \\ x_1+x_5=600 \end{cases}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 800 \\ 300 \\ 500 \\ 600 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 1 & 0 & 300 \\ 0 & 0 & 0 & 1 & 1 & 500 \\ 1 & 0 & 0 & 0 & 1 & 600 \end{bmatrix}$$

$-R_1+R_4$

$2+R_4$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 1 & 0 & 300 \\ 0 & 0 & 0 & 1 & 1 & 500 \\ 0 & -1 & 0 & 0 & 1 & -200 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 1 & 0 & 300 \\ 0 & 0 & 0 & 1 & 1 & 500 \\ 0 & 0 & -1 & 1 & 1 & 100 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 1 & 0 & 300 \\ 0 & 0 & -1 & 1 & 1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 500 \end{bmatrix} \xrightarrow[R_4+R_3]{-R_4+R_2} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 1 & 0 & 300 \\ 0 & 0 & 1 & -1 & -1 & -100 \\ 0 & 0 & 0 & 1 & 1 & 500 \end{bmatrix}$$

$$\begin{array}{c} \\ R_3+R_2 \\ \\ \\ \end{array}\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 0 & -1 & -200 \\ 0 & 0 & 1 & 0 & 0 & 400 \\ 0 & 0 & 0 & 1 & 1 & 500 \end{bmatrix} \xrightarrow{-R_2+R_1} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & 0 & 0 & -1 & 200 \\ 0 & 0 & 1 & 0 & 0 & 400 \\ 0 & 0 & 0 & 1 & 1 & 500 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 600 \\ 0 & 1 & 0 & 0 & -1 & 200 \\ 0 & 0 & 1 & 0 & 0 & 400 \\ 0 & 0 & 0 & 1 & 1 & 500 \end{bmatrix}$$

x_5 is free

$x_4 = 500 - x_5$

$x_3 = 400$

$x_2 = 200 + x_5$

$x_1 = 600 - x_5$

We can see the solution of this system depends on x_5 (which has to be smaller than 500).

* your book also includes another example about the solution of a system for finding equilibrium for an economy (the prices such that make the income for each sector match its expenditures.