1. (2 pts) Recall the minority connective $M$ where $\bar{v}(M(\varphi_1, \varphi_2, \varphi_3))$ agrees with the minority of $\bar{v}(\varphi_1), \bar{v}(\varphi_2), \bar{v}(\varphi_3)$. More precisely,

$$\bar{v}(M(\varphi_1, \varphi_2, \varphi_3)) = \begin{cases} 
1 & \text{iff } 2 \text{ or more of } \bar{v}(\varphi_1), \bar{v}(\varphi_2), \bar{v}(\varphi_3) \text{ are } 0 \\
0 & \text{iff } 2 \text{ or more of } \bar{v}(\varphi_1), \bar{v}(\varphi_2), \bar{v}(\varphi_3) \text{ are } 1
\end{cases}$$

Create a switching circuit using NOR gates which takes as inputs $A, B, C$ and outputs $M(A, B, C)$.

**Hint:** Write $M(A, B, C)$ is disjunctive normal form and use the following facts:

- $P \downarrow P$ is $\neg P$
- $P \lor Q$ is $(P \downarrow Q) \downarrow (P \downarrow Q)$

where $\downarrow$ is the connective representing NOR.

2. (1 pt each) For a formula $\varphi$ and a sentence symbol $A$ let $\varphi^\top_A$ and $\varphi^\bot_A$ be the formulas obtained from $\varphi$ by replacing $A$ with $\top$ and $\bot$ respectively. Then, let $\varphi^*_A$ be the formula $\varphi^\top_A \lor \varphi^\bot_A$. Prove the following:

(a) $\varphi \models \varphi^*_A$

(b) If $\varphi \models \psi$ and $A$ does not appear in $\psi$, then $\varphi^*_A \models \psi$.

(c) The formula $\varphi$ is satisfiable iff $\varphi^*_A$ is satisfiable.

3. (2 pts) Assume that every finite subset of $\Sigma$ is satisfiable. Let $\varphi$ be a formula. Show that one of the sets $\Sigma \cup \{\varphi\}$ or $\Sigma \cup \{\neg \varphi\}$ is also finitely satisfiable.