Research Statement

The main thrust of my research is in Asymptotic Geometric Analysis, closely linked to Combinatorics and Probability, a study which is often motivated by applications – signal processing, learning theory, complexity, data compression.

One example is the recent solution of the Talagrand's Entropy Problem, joint with S.Mendelson [MV]. Our positive solution to this conjecture has consequences in: *Combinatorics* (yielding a general form of the density theorem of Shelah, Sauer, Vapnik and Chervonenkis), *Probability* (proving a general Dudley's theorem on empirical processes known since 80's only in a particular case of $\{0, 1\}$ valued functions, and thus providing a direct alternative approach to the Glivenko-Cantelli Problem), *Functional Analysis* (extending to general spaces the theory of Bourgain and Tzafriri of invertibility of submatrices and proving the conjectured optimal Elton theorem, which completes the solution to a problem open since 1983), *Convex Geometry* (yielding sharp covering and volume estimates, hence a new information in the Milman's fundamental "Dvoretzky" Theorem), and *Applied Combinatorics* (solving one problem of N.Alon et al. on the Vapnik-Chernovenkis dimension, motivated by the Learning Theory). I believe, however, that the major part of the web of relationships and ideas still awaits its development.

Another example is a study of highly concentrated metric spaces, initiated by Gromov and Milman. Examples are many – expander graphs, all product spaces, Euclidean spheres, symmetric groups... The question is to find a simplest possible representation of those. Our characterization [RV] suggests that the Euclidean sphere must play an exceptional role among such spaces. This enabled us to generalize the embedding theorem of Bourgain-Lindenstrauss-Milman et al. from the Euclidean sphere to all such spaces [MV], but the final characterization is still awaited.

Some of the problems that currently interest me are – approximating matrices by random submatrices (I think we can substantially improve bounds in the approximation theorems of Alon, Kannan et al.); embedding finite metric spaces into l_1 (an open problem needed in data analysis); describing contact points of convex bodies (an isomorphic version of a problem of Tomczak-Jaegermann (1989) on contact points was settled in [V 01b]; an application of this study provides the optimal solution to a problem in communication networks [V 02]); discrete problems arising in Harmonic Analysis (the work [V 02] improves bounds of Bourgain-Tzafriri in the harmonic density problem), geometric properties of graphs; derandomization of the classical probabilistic inequalities (e.g. Khinchine's).