Homework 2

MAT 202, Winter 2004. Due 02/13/2004

1. (Closed subspaces) Let X be a normed space. Prove that every linear subspace of X is closed if and only if X is finite dimensional.

2. (Convergence and absolute convergence Let X be a normed space. Prove that X is a Banach space if and only if every absolutely convergent series of elements in X converges. (one direction is proved in class)

3. (Classical spaces) (i) Prove that $C(K)$ is a Banach space (K is a compact).

(ii) Prove that $L_1[0,1]$ with Lebesgue measure is a Banach space. Use Exercise 2 and classical convergence theorems on Lebesgue integral (such as the monotone convergence theorem).

(iii) Prove that $L_p[0,1]$ is a Banach space for $1 \leq p < \infty$.

4. (Functionals not attaining their norms) Prove that there exist continuous linear functionals for which the supremum in the definition of their norms is not attained. You may consider the functional f on $C[0,1]$ defined by $f(x) = \int_0^{1/2} x(t)\ dx - \int_{1/2}^1 x(t)\ dx$.