Final Exam

MAT 235C, Spring 2004. Due 06/11/2004 by 2pm in my mailbox

(the deadline is strict)

1. For finite irreducible Markov chains, prove or disprove each of the following statements:

(a) if the chain is aperiodic, then a stationary distribution exists;

(b) if a stationary distribution exists, then the chain is aperiodic;

(c) if a stationary distribution exists and every initial distribution converges to it ($\lim_n p_{ij}^{(n)} = \pi_j$ for all i, j), then the chain is aperiodic.

2. Prove or disprove the following statement: for every Markov chain $X_0, X_1, \dots$ and every real valued function f, the sequence $f(X_0), f(X_1), \dots$ also forms a Markov chain.

3. Suppose that we have a Markov chain on $S = \{0, 1, 2, ...\}$ such that $p_{00} = 1$ ("absorbing barrier") and $f_{i0} > 0$ for all i.

(a) Show that $\mathbb{P}_i(\exists j : \ X_n = j \text{ i.o.}) = 0$ for all i.

(b) Regard the state as the size of a population and interpret the conditions $p_{00} = 1$ and $f_{i0} > 0$ and the conclusion in part (a).

4. Consider an irreducible aperiodic Markov chain for which a stationary distribution exists. Show that the expected fraction of the time spent by the particle at a fixed state i converges to π_i in time. (First state this problem rigorously).