

Final Exam 21A on 12/13/02

Points: 200/

Name:

ID# :

This exam contains 8 pages. Check this. You are not allowed to use a calculator.

1. **Problem 1, 30pts:** Calculate the derivatives of the following functions with respect to x :

(a) [6pts] $\sin^{-1}\left(\frac{1}{x}\right)$;

(b) [6pts] $\frac{1}{\cos\left(\frac{1}{x}\right)}$;

(c) [6pts] $\ln\left(\frac{x}{x^2+1}\right)$;

(d) [6pts] $e^{-\cos^2 x}$;

(e) [6pts] $(\sqrt{x})^x$. (Hint: use logarithmic representation).

2. **Problem 2, 30pts:** Compute the following limits if they exist. (In case, explain why a limit does not exist).

(a) [6pts] $\lim_{x \rightarrow \infty} (x - \sqrt{1 + x^2})$;

(b) [6pts] $\lim_{x \rightarrow \infty} x e^{-x}$;

(c) [6pts] $\lim_{x \rightarrow 0} \frac{1}{\tan x}$;

(d) [6pts] $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(e) [6pts] $\lim_{x \rightarrow \pi/2} \frac{1}{\tan x}$.

3. **Problem 3, 50pts:** Let $f(x) = \frac{1}{2}x^2 + \frac{1}{x}$.

(a) [10pts] Find the domain and range of f ;

(b) [10pts] Find the region where f is increasing, respectively decreasing.

(c) [**10pts**] Compute the only local extremum, and decide whether it is a local maximum or minimum. Is that point a global extremum?

(d) [**10pts**] Find the region where f is concave upward, respectively concave downward.

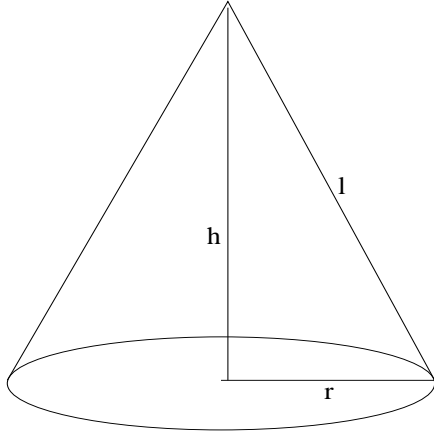
(e) [10pts] Plot the function qualitatively.

4. **Problem 4, 20pts:**

(a) [10pts] Show that there is a solution to the equation $x + 0.5 = 2^{-x}$. Provide reasons.

(b) [10pts] Calculate implicitly the derivative of $x + \tan(xy) = 2$ at $x = 1$.

5. **Problem 5, 30pts:** The volume V of a right circular cone with radius r and height h (see Figure) is equal to $\pi/3$. Find the radius **and** height for which the surface of the curved surface S is minimal. (Hint: you may use that $V = \frac{1}{3}r^2\pi h$, $S = \pi rl$.)



6. **Problem 6, 20pts:** Calculate

(a) [3pts] $\log_{10} 0.0001 =$

(b) [6pts] $\log_2 3 - \log_2 6 =$

(c) [5pts] $\ln \sqrt{e} =$

(d) [6pts] Express $\log_4 10$ in terms of $\log_2 10$.

7. **Problem 7, 20pts:** Calculate and plot the inverse function, f^{-1} , when

(a) [10pts] $f(x) = x^3 + 1$;

(b) [10pts] $f(x) = 3 \sin x$.