Final Exam 21A on 12/13/02

Points: 200/

Name: 
ID# :

This exam contains 8 pages. Check this. You are not allowed to use a calculator.

1. **Problem 1, 30pts**: Calculate the derivatives of the following functions with respect to $x$:
   (a) [6pts] $\sin^{-1}\left(\frac{1}{2}\right)$;
   
   (b) [6pts] $\frac{1}{\cos\left(\frac{1}{2}\right)}$;
   
   (c) [6pts] $\ln\left(\frac{x}{x^2+1}\right)$;
   
   (d) [6pts] $e^{-\cos^2x}$.
(e) [6pts] $(\sqrt{x})^x$. (Hint: use logarithmic representation).

2. **Problem 2, 30pts:** Compute the following limits if they exist. (In case, explain why a limit does not exist).

(a) [6pts] $\lim_{x \to \infty} \left( x - \sqrt{1 + x^2} \right)$;

(b) [6pts] $\lim_{x \to \infty} xe^{-x}$;

(c) [6pts] $\lim_{x \to 0} \frac{1}{\tan x}$;

(d) [6pts] $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.
(e) \([6\text{pts}] \lim_{x \to \pi/2} \frac{1}{\tan x}\).

3. **Problem 3, 50pts:** Let \(f(x) = \frac{1}{2}x^2 + \frac{1}{x}\).

(a) \([10\text{pts}]\) Find the domain and range of \(f\);

(b) \([10\text{pts}]\) Find the region where \(f\) is increasing, respectively decreasing.
(c) [10pts] Compute the only local extremum, and decide whether it is a local maximum or minimum. Is that point a global extremum?

(d) [10pts] Find the region where $f$ is concave upward, respectively concave downward.
(e) [10 pts] Plot the function qualitatively.

4. Problem 4, 20pts:
   
   (a) [10 pts] Show that there is a solution to the equation $x + 0.5 = 2^{-x}$. Provide reasons.

   (b) [10 pts] Calculate implicitly the derivative of $x + \tan(xy) = 2$ at $x = 1$. 


5. **Problem 5, 30pts:** The volume $V$ of a right circular cone with radius $r$ and height $h$ (see Figure) is equal to $\pi/3$. Find the radius and height for which the surface of the curved surface $S$ is minimal. (Hint: you may use that $V = \frac{1}{3}r^2\pi h, S = \pi rl$.)
6. **Problem 6, 20pts:** Calculate

(a) [3pts] \( \log_{10} 0.0001 = \)

(b) [6pts] \( \log_2 3 - \log_2 6 = \)

(c) [5pts] \( \ln \sqrt{e} = \)

(d) [6pts] Express \( \log_4 10 \) in terms of \( \log_2 10 \).
7. **Problem 7, 20pts**: Calculate and plot the inverse function, $f^{-1}$, when

(a) **[10pts]** $f(x) = x^3 + 1$;

(b) **[10pts]** $f(x) = 3 \sin x$. 
