Math 21A   Kouba
Practice Exam 2

1.) (6 pts. each) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = x \sqrt{4 - x^2}$

b.) $f(x) = \left(\dfrac{7 - x}{5x + 3}\right)^{10}$

c.) $y = 4\sin^2(x^3)$

d.) $g(x) = \pi^5 + \cos(\tan(\sec(\dfrac{3}{x})))$

2.) (12 pts.) Use $\lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$ to differentiate the function $f(x) = \sqrt{4 + x^2}$.

3.) You are standing on the top edge of a building which is 80 ft. high. You throw an egg straight DOWN at 64 ft./sec.
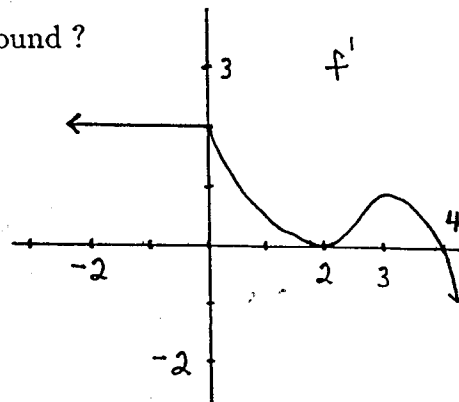
a.) (8 pts.) Assume that the acceleration due to gravity is $s''(t) = -32 ft./sec.^2$. Derive velocity, $s'(t)$, and height (above ground), $s(t)$, formulas for this doomed egg.

b.) (2 pts.) In how many seconds will the egg strike the ground ?

c.) (2 pts.) What is the egg's velocity as it strikes the ground ?

4.) (8 pts.) Sketch a possible graph of f using the graph

of its derivative, f'.

5.) (15 pts.) Consider the function $f(x) = (\dfrac{1}{4}) x^3 (4 - x)$ on the interval [-2, 5] . Determine where f is increasing, decreasing, concave up, and concave down. Identify all relative and absolute extrema, inflection points, and x- and y-intercepts. Sketch the graph. You may assume that $f'(x) = 3x^2 - x^3$ and $f''(x) = 6x - 3x^2$.

6.) a.) (4 pts.) State the Mean Value Theorem (MVT).

b.) (8 pts.) Determine if the following function satisfies the assumptions of the MVT. If so, find all values of c guaranteed by the conclusion of the MVT.

$f(x) = x + \sqrt{x - 2}$ on the interval [2, 6]

7.) (7 pts.) Assume that $s(t)$ is the distance in miles that a car travels in t hours. The derivative, $s'(t)$, represents the car's velocity. Why ?

8.) The graph given below represents the total shoe sales S (in \$) for a Davis shoe store during the first t months of 1996.

   a.) (2 pts.) Estimate the total shoe sales during the first 5 months of 1996 ?

   b.) (4 pts.) Estimate the total shoe sales between the first (1st) and fourth (4th) months of 1996 ?

   c.) (4 pts.) Estimate the RATE (in \$/month) of shoe sales when $t = 1$ month .
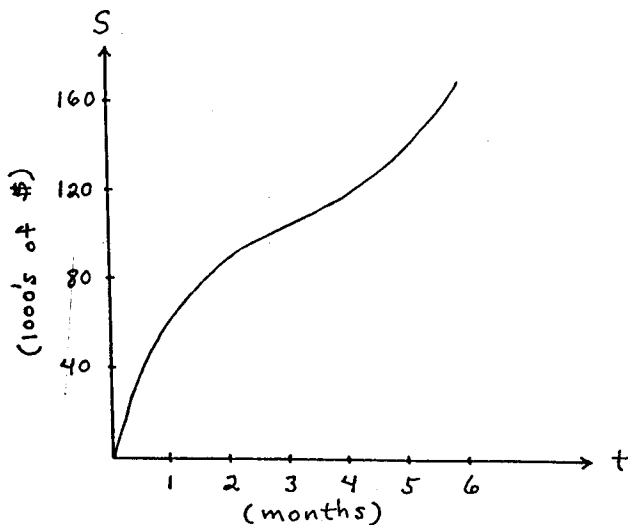
Each of the following two EXTRA CREDIT PROBLEMS is worth 8 points. These problems are OPTIONAL.

1.) Find all possible values for constants A and B so that the following function has a relative maximum at $x = 2$, or determine that this is impossible.
$f(x) = Ax^3 + B(x - 2)^2 + 12x$

2.) Use the limit definition of the derivative and appropriate trigonometry formulas to prove that $D \cos x = -\sin x$.