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Math 21A
Professor Vershynin
Fall 2006
Midterm 1

Problem	Points
1	/ 6
2	/ 9
3	/ 9
4	/ 5
5	/ 12
6	/ 7
7	/ 12
8	/ 10
9	/ 10
10	/ 8
11	/ 12
Total	/100

Please do not turn this page until told to do so.
No notes, books, or calculators may be used for this exam.
You must show ALL work to receive full credit on a problem.
You may NOT use L'Hopital's Rule for this exam.

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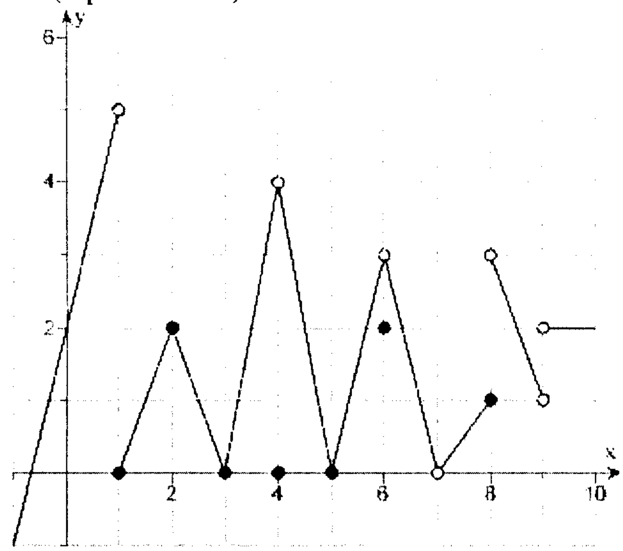
1. For the graph g(x) graphed below, find the following limits, if they exist. (2 points each)



a) $\lim_{x \to 2} g(x) =$ 2

b) $\lim_{x \to 8} g(x) =$ DNE

$\lim_{x \to 8^+} g(x) \neq \lim_{x \to 8^-} g(x)$

c) $\lim_{x \to 6} g(x) =$ 3

2. (9 points) Find the limit.

$$\lim_{h \to 0} \frac{\sqrt{19h+1}-1}{h} =$$

$$\lim_{h \to 0} \frac{\sqrt{19h+1}-1}{h} \cdot \frac{\sqrt{19h+1}+1}{\sqrt{19h+1}+1} = \lim_{h \to 0} \frac{19h+1-1}{h(\sqrt{19h+1}+1)}$$

$$= \lim_{h \to 0} \frac{19\cancel{h}}{\cancel{h}(\sqrt{19h+1}+1)} = \lim_{h \to 0} \frac{19}{\sqrt{19h+1}+1} =$$

$$\frac{19}{\sqrt{1}+1} = \frac{19}{2}$$

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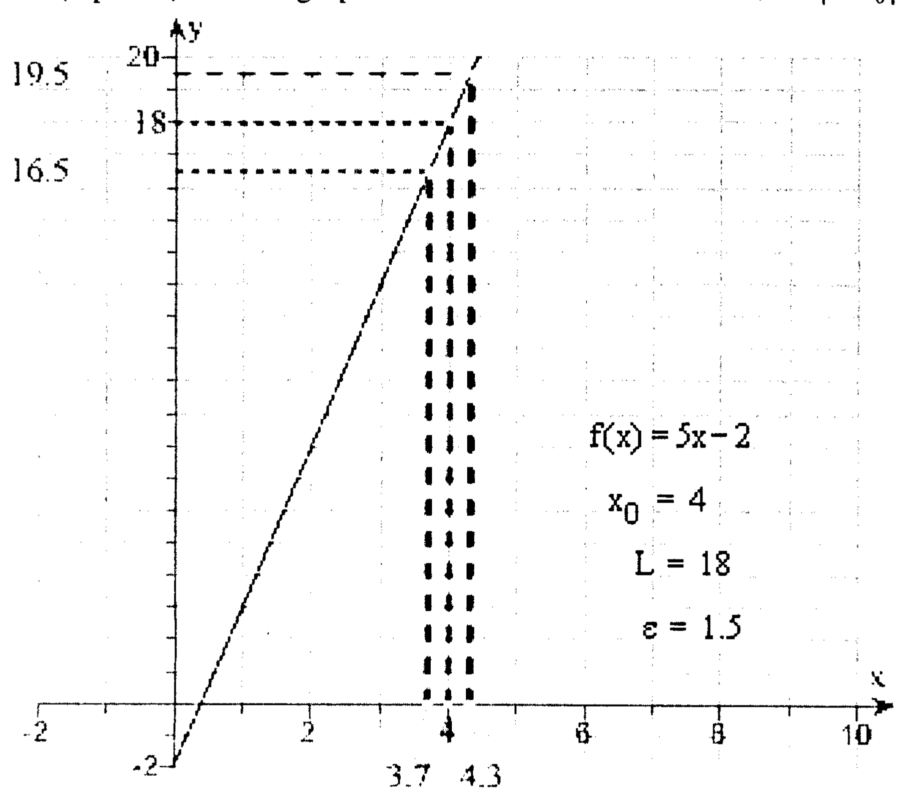
3. (9 points) Find the limit.

$$\lim_{x\to 144}\frac{\sqrt{x}-12}{x-144} = \lim_{x\to 144}\frac{\sqrt{x}-12}{(\sqrt{x}-12)(\sqrt{x}+12)}$$

$$= \lim_{x\to 144}\frac{1}{\sqrt{x}+12} = \frac{1}{\sqrt{144}+12} = \frac{1}{12+12} = \frac{1}{24}$$

4. (5 points) Use the graph to find δ>0 such that for all x, $0<|x-x_0|<\delta \Rightarrow |f(x)-L|<\varepsilon$



δ= 0.3

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5. (12 points) For the function f(x)= $\dfrac{x^2-169}{x-13}$, x_0=13 and ε=0.1

a) Find $L = \lim_{x \to x_0} f(x)$

$$\lim_{x \to 13} \frac{x^2-169}{x-13} = \lim_{x \to 13} \frac{(x+13)(x-\cancel{13})}{(x-\cancel{13})} = \lim_{x \to 13} x+13$$

$$= 13+13 = 26$$

b) Determine the largest value for δ>0 such that $0<|x-x_0|<\delta \Rightarrow |f(x)-L|<\varepsilon$

$$-.1 < \frac{x^2-169}{x-13} - 26 < .1$$

$$-.1 < x+13-26 < .1$$

$$-.1 < x-13 < .1$$

$$12.9 < x < 13.1$$

$$\delta = .1$$

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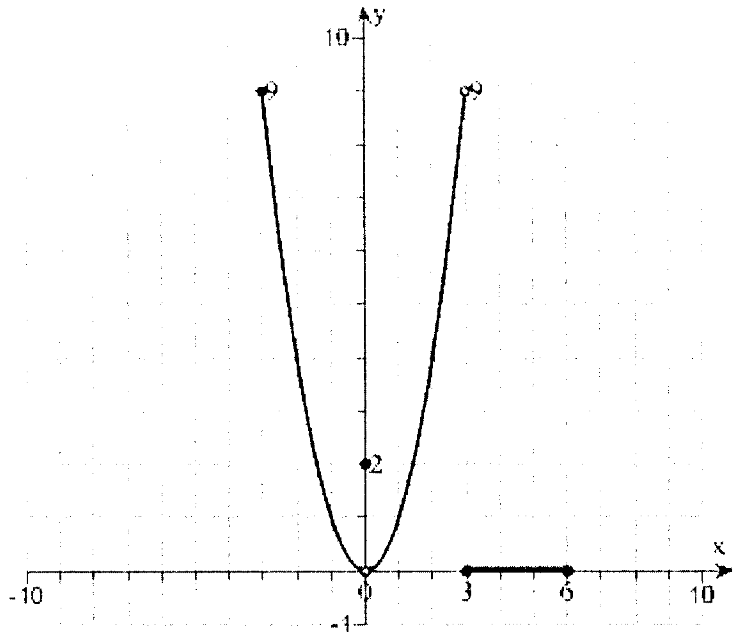
6. (1 point each) Use the graph below to determine whether the statement about the function y=f(x) is true or false. Circle T for true, and F for false



T / F $\lim_{x \to -2^-} f(x) = 4$

T / F $\lim_{x \to 0^-} f(x) = 1$

T / F $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

T / F $\lim_{x \to 0} f(x)$ exists

T / F $\lim_{x \to 0} f(x) = 0$

T / F $\lim_{x \to 2} f(x) = 4$

T / F $\lim_{x \to 4^-} f(x) = 4$

7. (12 points) Use the relation $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ to find the limit:

$$\lim_{x \to 0} \frac{\tan(10x)}{x} = \lim_{x \to 0} \frac{\frac{\sin 10x}{\cos 10x}}{x} = \lim_{x \to 0} \frac{\sin 10x}{x} \cdot \frac{1}{\cos 10x}$$

$$= 10 \lim_{x \to 0} \frac{\sin 10x}{10x} \cdot \frac{1}{\cos 10x}$$

$$= 10 \lim_{x \to 0} \frac{\sin 10x}{10x} \cdot \lim_{x \to 0} \frac{1}{\cos 10x}$$

$$= 10 (1) \cdot 1 = 10$$

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8. (9 points) Find the limit.
$\lim_{x\to\infty} 10e^{-x}\sin x$

1. $\lim_{x\to\infty} e^{-x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$

2. $-e^{-x} \le e^{-x}\sin x \le e^{-x}$

3. By Sandwich thm: $\lim_{x\to\infty} e^{-x}\sin x = 0$

Thus $\lim_{x\to\infty} 10e^{-x}\sin x = 10\lim_{x\to\infty} e^{-x}\sin x$

$= 10\cdot 0 = 0$

9. (10 points) Find the limit of f(x)=$\frac{7x+4}{9x+3}$ as

a) as x approaches ∞

$$\lim_{x\to\infty}\frac{7x+4}{9x+3} = \lim_{x\to\infty}\frac{(7x+4)\frac{1}{x}}{(9x+3)\frac{1}{x}} = \lim_{x\to\infty}\frac{7+\frac{4}{x}}{9+\frac{3}{x}}$$

$$= \frac{\lim_{x\to\infty} 7+\frac{4}{x}}{\lim_{x\to\infty} 9+\frac{3}{x}} = \frac{7}{9}$$

b) as x approaches $-\infty$

$$\lim_{x\to-\infty}\frac{7x+4}{9x+3} = \lim_{x\to-\infty}\frac{(7x+4)\frac{1}{x}}{(9x+3)\frac{1}{x}} = \lim_{x\to-\infty}\frac{7+\frac{4}{x}}{9+\frac{3}{x}}$$

$$= \frac{\lim_{x\to-\infty} 7+\frac{4}{x}}{\lim_{x\to-\infty} 9+\frac{3}{x}} = \frac{7}{9}$$

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10. (2 points each) Find the limits: $\lim \dfrac{1}{x^2 - 36}$

a) as $x \to 6^+$

$$\lim_{x \to 6^+} \frac{1}{x^2-36} = \lim_{x \to 6^+} \frac{1}{\underset{+}{(x-6)}\underset{+}{(x+6)}} = +\infty$$

b) as $x \to 6^-$

$$\lim_{x \to 6^-} \frac{1}{x^2-36} = \lim_{x \to 6^-} \frac{1}{\underset{-}{(x-6)}\underset{+}{(x+6)}} = -\infty$$

c) as $x \to -6^+$

$$\lim_{x \to -6^+} \frac{1}{x^2-36} = \lim_{x \to -6^+} \frac{1}{\underset{-}{(x-6)}\underset{+}{(x+6)}} = -\infty$$

d) as $x \to -6^-$

$$\lim_{x \to -6^-} \frac{1}{x^2-36} = \lim_{x \to -6^-} \frac{1}{\underset{-}{(x-6)}\underset{-}{(x+6)}} = +\infty$$

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11. (12 points) Find the horizontal and vertical asymptotes of f(x)=$\dfrac{4}{x-3}$, then graph the function.

a) Write the equation(s) of the horizontal asymptote(s) if any.

$$\lim_{x \to \infty} \frac{4}{x-3} = 0 \qquad \lim_{x \to -\infty} \frac{4}{x-3} = 0$$

Horizontal asymptote $y=0$

b) Write the equation(s) of the vertical asymptote(s) if any.

$$\lim_{x \to 3^+} \frac{4}{x-3} = +\infty \qquad \lim_{x \to 3^-} \frac{4}{x-3} = -\infty$$

Vert asymptote $x=3$

c) Graph the function.