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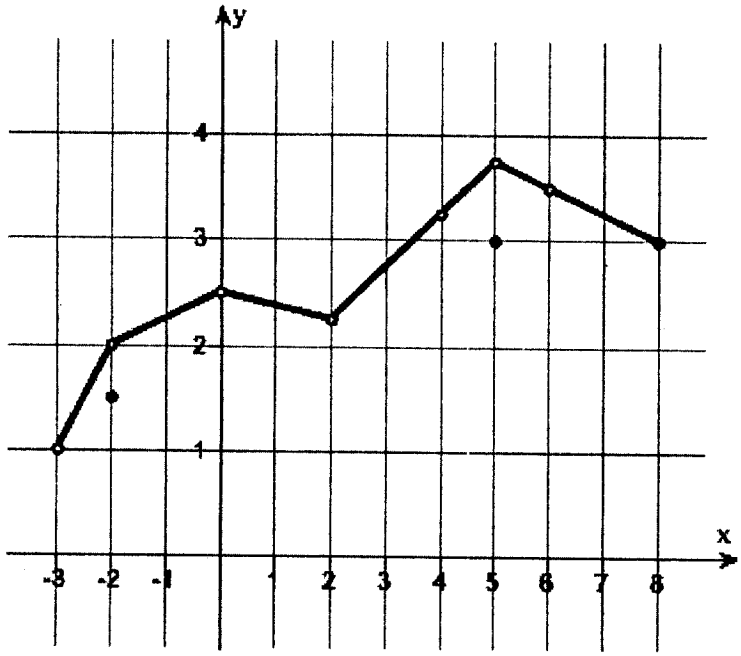
1. (10 points) Use the graph to identify the interval(s) on which f(x) is continuous



-f(x) is continuous on the interval(s):

$(-3,-2)\cup(-2,0)\cup(0,2)\cup(2,4)\cup(4,5)\cup(5,6)\cup(6,8]$

or

$(-3,8]$, $x\neq\{-2,0,2,4,5,6\}$

10

2. (10 points) Determine the point(s) at which the given function, f(x) is continuous:
f(x)=6csc(9x)
The function is continuous on (-∞,∞) except for:

10

$f(x)=6\cdot\frac{1}{\sin(9x)}=\frac{6}{\sin(9x)}$ +2

f(x) is not continuous when $\sin(9x)=0$ because f(x) will be undefined at that point +4

solve for critical points!

$\sin(9x)=0$

sin x is zero on $0\pm\pi k$

$9x=\arcsin(0)$

$9x=\pi k$, where k is an integer

$x=\frac{\pi}{9}k$

f(x) is continuous on $(-\infty,\infty)$ except for

$\boxed{x=\frac{\pi}{9}k}$, where k is an integer

+4

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3. (14 points) Find the derivative of $y = (x^2 - 7x + 7)e^{\frac{9x}{7}}$ and simplify your answer.

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$y' = (x^2-7x+7)' e^{\frac{9x}{7}} + (x^2-7x+7)(e^{\frac{9x}{7}})'$ +4

$y' = (2x-7)(e^{\frac{9x}{7}}) + (x^2-7x+7)(e^{\frac{9x}{7}})(\frac{9x}{7})'$

$y' = (2x-7)(e^{9x/7}) + (x^2-7x+7)(e^{9x/7})(\frac{9}{7})$ +6

$y' = (e^{9x/7})\left[(2x-7) + \frac{9}{7}(x^2-7x+7)\right]$

$y' = e^{9x/7}\left(2x-7 + \frac{9}{7}x^2 - 9x + 9\right)$

$\boxed{y' = e^{9x/7}\left(\frac{9}{7}x^2 - 7x + 2\right)}$ +4

4. (15 points) Find the derivative of $f(x) = \left(\frac{\cos x}{1-\sin x}\right)^2$ and simplify your answer

$f'(x) = 2\left(\frac{\cos x}{1-\sin x}\right)\left(\frac{\cos x}{1-\sin x}\right)'$

$f'(x) = 2\left(\frac{\cos x}{1-\sin x}\right)\left[\frac{(\cos x)'(1-\sin x) - (\cos x)(1-\sin x)'}{(1-\sin x)^2}\right]$

$f'(x) = 2\left(\frac{\cos x}{1-\sin x}\right)\left[\frac{(-\sin x)(1-\sin x) - (\cos x)(-\cos x)}{(1-\sin x)^2}\right]$

$f'(x) = 2\left(\frac{\cos x}{1-\sin x}\right)\left[\frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}\right]$

$f'(x) = 2\left(\frac{\cos x}{1-\sin x}\right)\left[\frac{-\sin x + (\sin^2 x + \cos^2 x)}{(1-\sin x)^2}\right]$

$f'(x) = 2\left(\frac{\cos x}{1-\sin x}\right)\left[\frac{-\sin x + 1}{(1-\sin x)^2}\right]$

$f'(x) = 2\left(\frac{\cos x}{1-\sin x}\right)\left[\frac{1-\sin x}{(1-\sin x)^2}\right]$

$f'(x) = 2\left(\frac{\cos x}{1-\sin x}\right)\left(\frac{1}{1-\sin x}\right) = 2\left[\frac{\cos x}{(1-\sin x)^2}\right] = \boxed{\frac{2\cos x}{(1-\sin x)^2}}$

+15

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5. (14 points) Find the slope of the curve at the given point: $4y^9 + 5x^7 = 7y + 2x$ at (1,1)

$$\frac{d}{dx}\left[4y^9 + 5x^7\right] = \frac{d}{dx}\left[7y + 2x\right]$$

$$36y^8\frac{dy}{dx} + 35x^6 = 7\frac{dy}{dx} + 2$$

$$36y^8\frac{dy}{dx} - 7\frac{dy}{dx} = 2 - 35x^6$$

$$\frac{dy}{dx}(36y^8 - 7) = 2 - 35x^6$$

$$\frac{dy}{dx} = \frac{2 - 35x^6}{36y^8 - 7}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=1}} = \frac{2 - 35(1)^6}{36(1)^8 - 7} = \frac{2-35}{36-7} = \boxed{\frac{-33}{29}}$$ ✓

+14

6. (13 points) Use logarithmic differentiation to find the derivative of $y = (t+9)(t+8)(t+5)$

$$\ln y = \ln\left[(t+9)(t+8)(t+5)\right]$$

$$\ln y = \ln(t+9) + \ln(t+8) + \ln(t+5)$$

$$\frac{d}{dt}\left[\ln y\right] = \frac{d}{dt}\left[\ln(t+9) + \ln(t+8) + \ln(t+5)\right]$$

$$\frac{1}{y}\cdot\frac{dy}{dt} = \frac{1}{t+9}(t+9)' + \frac{1}{t+8}(t+8)' + \frac{1}{t+5}(t+5)'$$

$$\frac{1}{y}\frac{dy}{dt} = \frac{1}{t+9}(1) + \frac{1}{t+8}(1) + \frac{1}{t+5}(1)$$

$$\frac{dy}{dt} = y\left[\frac{1}{t+9} + \frac{1}{t+8} + \frac{1}{t+5}\right]$$

$$\frac{dy}{dt} = (t+9)(t+8)(t+5)\left[\frac{1}{t+9} + \frac{1}{t+8} + \frac{1}{t+5}\right]$$ ✓

+13

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7. (12 points) Find the linearization L(x) at x=-1 of $f(x) = -x - \frac{1}{x}$

$f(x) = -x - x^{-1}$

$f'(x) = (-x)' + (-x^{-1})'$

$f'(x) = -1 + x^{-2} = -1 + \frac{1}{x^2}$

$f'(-1) = -1 + \frac{1}{(-1)^2} = -1 + 1 = 0$

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$L(x) = f(a) + f'(a)(x-a)$

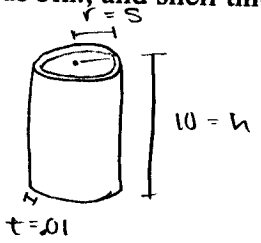
$L(x) = -a - \frac{1}{a} + \left(-1 + \frac{1}{a^2}\right)(x-a)$

$L(-1) = 1 - \frac{1}{-1} + (0)(x+1)$

$= 2$

$L(x) = 2$ horizontal line w/ slope 0

8. (12 points) Find the volume of material in a cylindrical shell with height 10in. radius 5in., and shell thickness .01in.



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V_1 = volume of cylinder

V_2 = volume of inner cylindrical shell

$V_1 - V_2$ = volume of cylindrical shell

$V_1 = \pi r^2 h$

$V_2 = \pi (r - .01)^2 h$

$V = V_1 - V_2$

$V = \pi r^2 h - \pi (r - .01)^2 h$

$V = \pi (5)^2 (10) - \pi (5 - .01)^2 (10)$

$V = 250\pi - \pi (4.99)^2 (10)$

$V = 250\pi - \pi (24.9001)(10)$

$V = 250\pi - 249.001\pi$

$\boxed{V = 0.999\pi}$

$$\begin{array}{r} 4.99 \\ \times 4.99 \\ \hline 4491 \\ 44910 \\ 199600 \\ \hline 24.9001 \end{array}$$