Math 21B

Final Exam

Please do not turn this page until told to do so. No notes, books, or calculators may be used for this exam. You must show NEAT and COMPLETE work to receive full credit on a problem.

Name: Answer Key

Section: ____________________

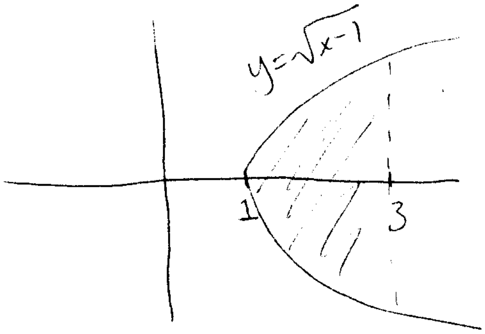
SID: ____________________

Signature: ____________________

Sections:
001: Deanna, 7:10-8:10
002: Mo, 5:10-6:10
003: Mihaela, 6:10-7:10
004: Josh, 8:10-9:10

Problem(s)	Score
1	/ 15
2	/ 10
3	/ 15
4	/ 15
5	/ 15
6	/ 15
7	/ 15
Total	/ 100

(15 pts) **1.** Consider the solid generated by revolving the region bounded on the left by the parabola $x = y^2 + 1$ and on the right by the line $x = 3$ about the x-axis. Find the volume of the solid.



(Disk Method)

$$V = \int_1^3 \pi R(x)^2 dx = \pi \int_1^3 \left(\sqrt{x-1}\right)^2 dx$$

$$= \pi \int_1^3 (x-1)\, dx = \pi \left[\frac{x^2}{2} - x \right]_1^3$$

$$= \pi \left[\left(\frac{9}{2} - 3\right) - \left(\frac{1}{2} - 1\right) \right] = \boxed{2\pi}$$

(10 pts) **2.** Find the length of the parametrized curve

$$x = e^t - t, \quad y = 4e^{t/2}, \quad 0 \le t \le 1.$$

$$\frac{dx}{dt} = e^t - 1 \qquad \frac{dy}{dt} = 2e^{t/2} \quad \Rightarrow$$

$$\sqrt{(dx/dt)^2 + (dy/dt)^2} = \sqrt{(e^t - 1)^2 + (2e^{t/2})^2}$$

$$= \sqrt{e^{2t} + 2e^t + 1} = \sqrt{(e^t + 1)^2} = e^t + 1 \Rightarrow$$

$$L = \int_0^1 (e^t + 1)\, dt = \left[e^t + t \right]_0^1 = (e+1) - 1 = \boxed{e}$$

(15 pts) **3.** Evaluate the integral

$$\int x^2 \ln x \, dx.$$

(By Parts) $u = \ln x$ $dv = x^2 dx$

$\Rightarrow$ $du = dx/x$, $v = x^3/3$

Therefore, $\int x^2 \ln x \, dx = \dfrac{x^3 \ln x}{3} - \int \dfrac{x^3}{3} \cdot \dfrac{dx}{x} = \dfrac{x^3 \ln x}{3} - \dfrac{1}{3}\int x^2 dx$

$= \dfrac{x^3 \ln x}{3} - \dfrac{x^3}{9} + C$ or $\dfrac{x^3}{3}\left(\ln x - \dfrac{1}{3}\right) + C$

(15 pts) **4.** Evaluate the integral

$$\int_0^{\pi/2} \sin(y)\, e^{\cos y} \, dy.$$

$u = \cos(y)$ $du = -\sin y \, dy$

Bounds: $y = 0 \Rightarrow u = 1$; $y = \pi/2 \Rightarrow u = 0$.

Therefore, $\int_0^{\pi/2} \sin(y)\, e^{\cos y} dy = -\int_1^0 e^u du = \int_0^1 e^u du$

$= \left[e^u\right]_0^1 = \boxed{e - 1}$

(15 pts) 5. Evaluate the integral

$$\int \frac{x\,dx}{x-4}.$$

$$\frac{x}{x-4} = \frac{x-4+4}{x-4} = 1 + \frac{4}{x-4}$$

So,

$$\int \frac{x\,dx}{x-4} = \int \left(1 + \frac{4}{x-4}\right) dx$$

$$= \boxed{x + 4\ln|x-4| + C}$$

(15 pts) 6. Evaluate the integral

$$\int \frac{x\,dx}{x^2-3x+2}.$$

(Partial Fractions)

$x^2-3x+2 = (x-1)(x-2)$, so $\dfrac{x}{x^2-3x+2} = \dfrac{A}{x-1} + \dfrac{B}{x-2}$

$\Rightarrow \quad x = A(x-2) + B(x-1)$

$\Rightarrow \begin{cases} A+B=1 \\ 2A+B=0 \end{cases} \Rightarrow \quad A=-1,\ B=2$

$$\Rightarrow \int \frac{x\,dx}{x^2-3x+2} = -\int \frac{dx}{x-1} + 2\int \frac{dx}{x-2} =$$

$$-\ln|x-1| + 2\ln|x-2| + C$$

(15 pts) **7.** An alluminum beam was brought from the outside cold into a machine shop where the temperature was held at 65°F. After 10 min. the beam warmed to 35°F and after another 10 min it was 50°F. Find the beam's initial temperature.

$$T - 65 = (T_0 - 65)e^{-kt}, \quad \text{so}$$

$$\begin{cases} 35 - 65 = (T_0 - 65)e^{-10k} \\ 50 - 65 = (T_0 - 65)e^{-20k} \end{cases} \Rightarrow \begin{cases} -30 = (T_0 - 65)e^{-10k} \\ -15 = (T_0 - 65)e^{-20k} \end{cases}$$

$$\Rightarrow \cancel{(T_0 - 65)}e^{-10k} = 2\cancel{(T_0 - 65)}e^{-20k}$$

$$\Rightarrow e^{10k} = 2$$

$$\Rightarrow k = \frac{\ln 2}{10}.$$ Substitute into

$$\Rightarrow -30 = (T_0 - 65)\cdot\frac{1}{2}$$

$$\Rightarrow \boxed{T_0 = 5°F}$$