1

$$\frac{dy}{dx} = e^{-x} \Rightarrow y = -e^{-x} + C;$$

at $x = 0$ and $y = 3$ we have $3 = -1 + C$

$$\Rightarrow C = 4 \Rightarrow \boxed{y = -e^{-x} + 4}$$

$$\cancel{\int \sqrt{x}\left(3 + \frac{1}{x}\right)dx = \int \left(3\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx = 3\int x^{1/2}\,dx + \int x^{-1/2}\,dx}$$

$$\cancel{= 3 \cdot \frac{2}{3} x^{3/2} + 2x^{1/2} + C = \boxed{2\sqrt{x}(x+1) + C}.}$$

2

Solution 1: $f(x) = (\sin x)^7 \cos x$ is an odd function.

Indeed, $f(-x) = (\sin(-x))^7 \cos(-x) = (-\sin x)^7 \cos x = -(\sin x)^7 \cos x$

$= -f(x)$.

Therefore, $\int_{-\pi/2}^{\pi/2} f(x)\,dx = \boxed{0}$.

Solution 2: Substitution $u = \sin x$.

$\Rightarrow du = \cos x\,dx$

When $x = -\frac{\pi}{2}$, $u = \sin\left(-\frac{\pi}{2}\right) = -1$

When $x = \frac{\pi}{2}$, $u = \sin\left(\frac{\pi}{2}\right) = 1$.

$$\int_{-1}^{1} u^7\,du = \left.\frac{u^8}{8}\right]_{-1}^{1} = \frac{1}{8} - \frac{1}{8} = \boxed{0}$$

(3)

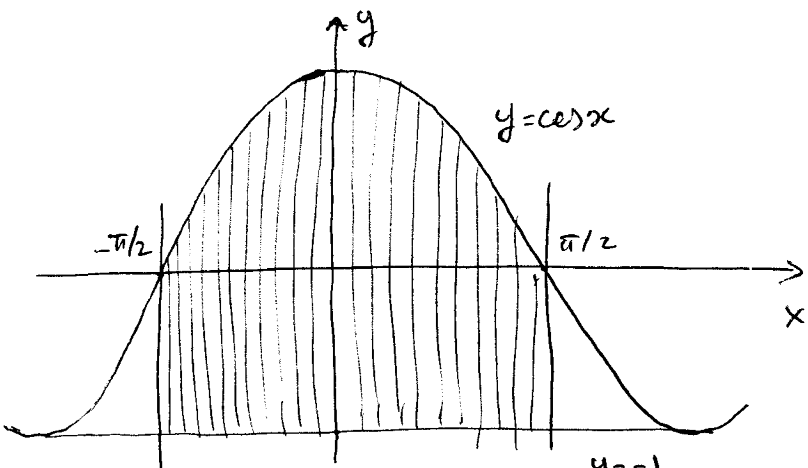
Substitution $u = \ln x \Rightarrow du = \frac{1}{x}dx$.

When $x = 3$, $u = \ln 3$; when $u = 9$, $u = \ln 9$.

$$\int_{\ln 3}^{\ln 9} \frac{du}{u} = \ln u \Big]_{\ln 3}^{\ln 9} = \ln(\ln 9) - \ln(\ln 3) = \ln\left(\frac{\ln 9}{\ln 3}\right)$$

$$= \ln\left(\frac{\ln 3^2}{\ln 3}\right) = \ln\left(\frac{2\ln 3}{\ln 3}\right) = \boxed{\ln 2}.$$

(4)



$$A = \int_{-\pi/2}^{\pi/2} (\cos x - (-1))\, dx$$

$$= 2\int_0^{\pi/2} (\cos x + 1)\, dx \quad \text{(the function is even)}$$

$$= 2(\sin x + x)\Big]_0^{\pi/2}$$

$$= 2\left(\sin\left(\frac{\pi}{2}\right) + \frac{\pi}{2}\right) = 2\left(1 + \frac{\pi}{2}\right)$$

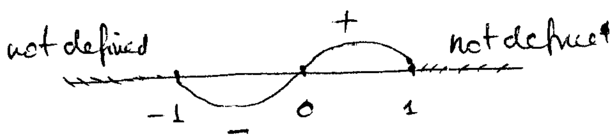
$$= \boxed{\pi + 2}.$$

Zeros of $f(x) = x\sqrt{1-x^2}$:

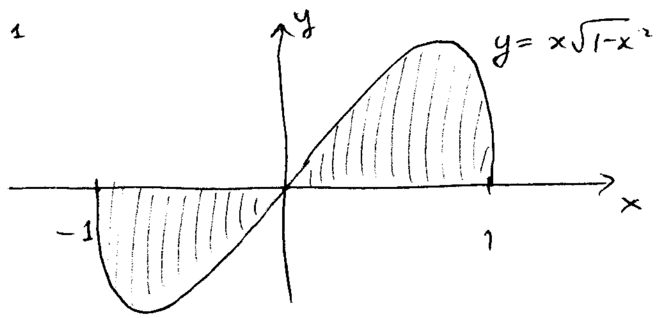
$$x\sqrt{1-x^2} = 0 .$$

$$x = 0 \quad \text{or} \quad 1 - x^2 = 0$$

$$x = 0 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 1 .$$



Sketch of the graph :



The function is <u>odd</u>, so the area A consists of two identical quantities :

$$A = 2\int_0^1 x\sqrt{1-x^2}\,dx .$$

Substitution : $u = 1 - x^2 .$ $\Rightarrow$ $du = -2x\,dx .$

When $x = 0,\ u = 1 .$ When $x = 1,\ u = 0 .$

$$A = \int_1^0 \left(-\sqrt{u}\right) du = \int_0^1 \sqrt{u}\,du = \int_0^1 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big]_0^1 = \boxed{\frac{2}{3}} .$$