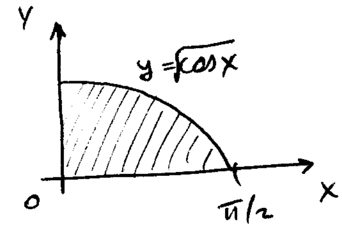
MIDTERM 2 Solutions

1

$R(x) = \sqrt{\cos x} \Rightarrow V = \int_0^{\pi/2} \pi (R(x))^2 \, dx$



$= \pi \int_0^{\pi/2} \cos x \, dx = \pi \Big[\sin x\Big]_0^{\pi/2}$

$= \pi(1-0) = \boxed{\pi.}$

2

$\frac{dx}{dt} = 8t\cos t, \quad \frac{dy}{dt} = 8t\sin t. \Rightarrow$

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{64t^2\cos^2 t + 64t^2\sin^2 t} = 8|t|\sqrt{\cos^2 t + \sin^2 t}$

$= 8t$ since $t \geq 0$.

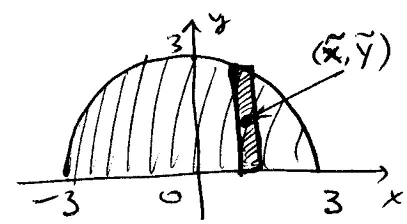
$\text{length} = \int_0^{\pi/2} 8t \, dt = 4t^2\Big]_0^{\pi/2} = \boxed{\pi^2}.$

4

~~Applying~~

Because the density is constant and the region is symmetric about x-axis, we conclude that

$\bar{x} = 0$.



Using the typical vertical strip as shown here, we have:

$(\tilde{x}, \tilde{y}) = \left(x, \frac{\sqrt{9-x^2}}{2}\right).$

Width of strip: dx.

length $= \sqrt{9-x^2}$.

Area : $dA = \sqrt{9-x^2}\,dx$.

Mass : $dm = \delta\, dA = \delta\sqrt{9-x^2}\,dx$.

Therefore $M_x = \int_{-3}^{3} \tilde{y}\, dm = \int_{-3}^{3} \frac{\sqrt{9-x^2}}{2} \cdot \delta\sqrt{9-x^2}\,dx$

$$= \int_{-3}^{3} \frac{\delta}{2}(9-x^2)\,dx \underset{\substack{\uparrow \\ \text{by symmetry}}}{=} \delta\int_{0}^{3}(9-x^2)\,dx = 18\delta.$$

$M = \delta \cdot A$, where A is the area of the semicircle $\Rightarrow$

$A = \frac{1}{2}(\pi \cdot 3^2) = \frac{9\pi}{2}$.

Hence $M = \frac{9\pi\delta}{2}$.

Therefore, $\bar{y} = \frac{M_x}{M} = \frac{18\delta}{9\pi\delta/2} = \boxed{\frac{4}{\pi}}$.

Answer: $\boxed{\left(0, \frac{4}{\pi}\right) \text{ is the center of mass.}}$

3

Force constant ? $F = kx$

$20 = k \cdot 1$

$\Rightarrow k = 20$ lb/ft.

The work to strech the spring 1 ft is

$$W = \int_0^1 kx\,dx = k\int_0^1 x\,dx = 20 \cdot \frac{1}{2} = \boxed{10 \text{ ft}\cdot\text{lb}.}$$