MAT 21B-A, Spring 2007, Prof. Opmeer
Wednesday May 30, 2007.

# MIDTERM EXAM 3

NAME (print in CAPITAL LETTERS, last name first):

Solutions

ID#:

**Instructions:**

- Read each question carefully and answer in the space provided. If the space provided is not enough, please continue on the opposite (blank) side and CLEARLY INDICATE this.
- YOU MUST GIVE CLEAR AND REASONABLY COMPLETE ANSWERS TO RECEIVE FULL CREDIT. Answers like 'yes' or 'no' or '2' will be awarded no credit. Proper notation and (mathematical) readability of your answers play a role in determining credit.
- Calculators, books, notes or similar things are not allowed.
- The numbers between brackets refer to the number of points (out of 100) for that exercise.

1.[100] Compute the following indefinite integrals.
Indicate the substitution that you make (if you make one) and state the trigonometric identity that you use (if you use one).

(a)[10] $\int \frac{16x}{8x^2+1}\,dx$

$u = 8x^2+1$

$du = 16x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \boxed{\ln|8x^2+1| + C}$

(b)[10] $\int \frac{1}{x^2+2x}\,dx$

$= \int \frac{1}{x(x+2)}\,dx$

$= \int \left(\frac{1/2}{x} + \frac{-1/2}{x+2}\right)dx$

$= \boxed{\frac{1}{2}\ln|x| - \frac{1}{2}\ln|x+2| + C}$

$\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$1 = A(x+2) + Bx$

$\underline{x=0}$: $1 = 2A \Rightarrow A = 1/2$

$\underline{x=-2}$: $1 = -2B \Rightarrow B = -1/2$

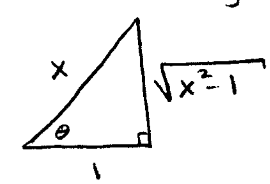
$\tan^2\theta = \sec^2\theta - 1$

(c)[10] $\int \frac{1}{(x^2-1)^{3/2}}\,dx,\ x > 1$

$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\sec\theta = \frac{x}{1} = \frac{\text{hyp.}}{\text{adj.}}$




$$= \int \frac{1}{(\sec^2\theta - 1)^{3/2}} \cdot \sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{(\tan^2\theta)^{3/2}} \cdot \sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sec\theta\tan\theta}{\tan^3\theta}\,d\theta$$

$$= \int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= \int \frac{1}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$= \int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \int \frac{1}{u^2}\,du$$

$$= -\frac{1}{u} + C$$

$$= -\frac{1}{\sin\theta} + C = \boxed{-\frac{x}{\sqrt{x^2-1}} + C}$$

(d)[10] $\int \frac{\ln\sqrt{x}}{x}\,dx$

$$= \int \frac{\frac{1}{2}\ln x}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \frac{1}{2}\int \frac{\ln x}{x}\,dx$$

$$= \frac{1}{2}\int u\,du$$

$$= \frac{1}{2}\cdot\frac{1}{2}u^2 + C$$

$$= \boxed{\frac{1}{4}(\ln x)^2 + C}$$

(e)[10] $\int \frac{8}{x^2-2x+2}\,dx$

$$= \int \frac{8}{(x-1)^2+1}\,dx$$

$u = x-1$

$du = dx$

$$= \int \frac{8}{u^2+1}\,du$$

$$= 8\arctan u + C$$

$$= \boxed{8\arctan(x-1) + C}$$

$\cos^2 x = \frac{1+\cos 2x}{2}$ , $\sin^2 x = \frac{1-\cos 2x}{2}$

$1-\cos^2\theta = \sin^2\theta$

(f)[10] $\int 8\sin^4 x\cos^2 x\,dx$

$$= 8\int \left(\frac{1-\cos 2x}{2}\right)^2\left(\frac{1+\cos 2x}{2}\right)dx$$

$$= \int (1-\cos^2 2x)(1+\cos 2x)\,dx$$

$$= \int \sin^2 2x\,(1+\cos 2x)\,dx$$

$$= \int \sin^2 2x\,dx + \int \sin^2 2x\cos 2x\,dx$$

$u = \sin 2x$

$du = 2\cos 2x\,dx$

$$= \int \frac{1-\cos 4x}{2}\,dx + \frac{1}{2}\int u^2\,du$$

$$= \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right] + \frac{1}{2}\cdot\frac{u^3}{3} + C$$

$$= \boxed{\frac{1}{2}x - \frac{\sin 4x}{8} + \frac{1}{6}\sin^3 2x + C}$$

<u>Note</u>: This is not the only way this could be done!

$$\int u\,dv = uv - \int v\,du$$

(g)[10] $\int x^2 \cos x \, dx$

$$\begin{array}{ll} u = x^2 & dv = \cos x\,dx \\ du = 2x\,dx & v = \sin x \\ \hline u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{array}$$

$$= x^2 \sin x - \int 2x \sin x \, dx$$

$$= x^2 \sin x - 2\left[x(-\cos x) - \int -\cos x\,dx\right]$$

$$= \boxed{x^2 \sin x - 2\left[-x\cos x + \sin x\right] + C}$$

(h)[10] $\int \sqrt{25 - x^2}\, dx$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$(5\cos\theta)^2 = 25 - (5\sin\theta)^2$$

$$\boxed{\begin{array}{l} x = 5\sin\theta \\ dx = 5\cos\theta\, d\theta \end{array}}$$

$$\sin\theta = \frac{x}{5} = \frac{\text{opp.}}{\text{hyp.}}$$

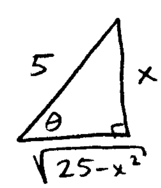
$$= \int \sqrt{25 - 25\sin^2\theta} \cdot 5\cos\theta\, d\theta$$

$$= \int 5\cos\theta \cdot 5\cos\theta \, d\theta$$

$$= 25 \int \cos^2\theta \, d\theta$$

$$= 25 \int \left(\frac{1 + \cos 2\theta}{2}\right) d\theta$$

$$= \frac{25}{2}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$




$$\frac{\sin 2\theta}{2} = \frac{\cancel{2} \sin\theta\cos\theta}{\cancel{2}}$$

$$= \boxed{\frac{25}{2}\left[\arcsin\left(\frac{x}{5}\right) + \left(\frac{x}{5}\right)\left(\frac{\sqrt{25-x^2}}{5}\right)\right] + C}$$

(i)[10] $\int \arctan x\, dx$

$$u = \tan^{-1} x \qquad dv = dx$$
$$du = \frac{1}{x^2+1}\,dx \qquad v = x$$

$$= x\tan^{-1}x - \int \frac{x}{x^2+1}\,dx$$

$$w = x^2+1$$
$$dw = 2x\,dx$$

$$= x\tan^{-1}x - \frac{1}{2}\int \frac{1}{w}\,dw$$

$$= x\tan^{-1}x - \frac{1}{2}\ln|w| + C$$

$$= \boxed{x\tan^{-1}x - \frac{1}{2}\ln|x^2+1| + C}$$

(j)[10] $\int \frac{3x+2}{x^2+2x+1}\,dx$

$$\frac{3x+2}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$3x+2 = A(x+1) + B$$

$\underline{x=-1}$: $\quad -1 = B \qquad B=-1$

$\underline{x=0}$: $\quad 2 = A+B$

$$2 = A - 1 \Rightarrow A = 3$$

$$= \int \frac{3}{x+1}\,dx + \int \frac{-1}{(x+1)^2}\,dx$$

$$u = x+1$$
$$du = dx$$

$$= 3\ln|x+1| - \int u^{-2}\,du$$

$$= 3\ln|x+1| + u^{-1} + C$$

$$= \boxed{3\ln|x+1| + (x+1)^{-1} + C}$$