1. **Discrete independent random variables**
   (a) Show that if $X$ and $Y$ are independent, integer valued random variables, then
   \[ P(X + Y = n) = \sum_k P(X = k)P(Y = n - k). \]

   (b) Let $X$ and $Y$ be independent Poisson random variables with parameters $\lambda$ and $\mu$ respectively. Prove that $X + Y$ is a Poisson random variable with parameter $\lambda + \mu$.

   (c) Let $X$ and $Y$ be independent Binomial random variables with parameters $(n, p)$ and $(m, p)$ respectively. Prove that $X + Y$ is a Binomial random variable with parameter $(n + m, p)$.

2. **Uncorrelated but not independent random variables**
   Consider the probability space $\Omega = [-1/2, 1/2]$ with the normalized Lebesgue measure on it. Construct two random variables $X, Y$ on $\Omega$ that are uncorrelated but not independent. (Hint: consider $X(x) = x, Y(x) = ax^2 + b$).

3. **Product of independent random variables**
   Let $X, Y > 0$ be independent random variables with distribution functions $F$ and $G$.
   (a) Find the distribution function of $XY$.
   (b) Compute the distribution function of $XY$ if $X$ and $Y$ are independent random variables uniformly distributed on $[0, 1]$.

4. **Convergence in probability and convergence in $L^p$**
   We have shown in class that convergence in $L^p$ implies convergence in probability for all $p > 0$. Show that the converse does not hold. (Prove this by example for $p = 1$).

5. **The coupon collector problem**
   (a) How many times does one need to toss a coin (on average) until the first head occurs?
   (b) How many times does one need to roll a dice (on average) until the first “six dots” occurs?
   (c) How many times does one need to roll a dice (on average) until all faces have appeared at least once?
   (d) Each time one buys a bag of cheese doodles there is one bonus coupon inside. There are $n$ different coupons that are equally likely to be inside any bag. Prove that one needs to buy about $Cn \log n$ bags on average in order to have a complete collection of the coupons. (Here $C$ is some constant).