Homework 6

Probability Theory (MATH 235A, Fall 2007)

1. Discrete independent random variables (a) Show that if X and Y are independent, integer valued random variables, then

$$\mathbb{P}(X+Y=n)=\sum_k \mathbb{P}(X=k)\,\mathbb{P}(Y=n-k).$$

(b) Let X and Y be independent Poisson random variables with parameters λ and μ respectively. Prove that $X+Y$ is a Poisson random variable with parameter $\lambda+\mu$.

(c) Let X and Y be independent Binomial random variables with parameters (n,p) and (m,p) respectively. Prove that $X+Y$ is a Binomial random variable with parameter $(n+m,p)$.

2. Uncorrelated but not independent random variables Consider the probability space $\Omega=[-1/2,1/2]$ with the normalized Lebesgue measure on it. Construct two random variables X, Y on Ω that are uncorrelated but not independent. (Hint: consider $X(x)=x$, $Y(x)=ax^2+b$).

3. Product of independent random variables Let $X,Y>0$ be independent random variables with distribution functions F and G.

(a) Find the distribution function of XY.

(b) Compute the distribution function of XY if X and Y are independent random variables uniformly distributed on $[0,1]$.

4. Convergence in probability and covergence in L^p We have shown in class that convergence in L^p implies convergence in probability for all $p>0$. Show that the converse does not hold. (Prove this by example for $p=1$).

5. The coupon collector problem (a) How many times does one need to toss a coin (on average) until the first head occurs?

(b) How many times does one need to roll a dice (on average) until the first "six dots" occurs?

(c) How many times does one need to roll a dice (on average) until all faces have appeared at least once?

(d) Each time one buys a bag of cheese doodles there is one bonus coupon inside. There are n different coupons that are equally likely to be inside any bag. Prove that one needs to buy about $Cn\log n$ bags on average in order to have a complete collection of the coupons. (Here C is some constant).