QUIZ Solutions

(1)

Let H mean heads, T mean tails.

That the experiment ends before the sixth toss means that we get one of the following sequences:

HH, TT

HTT, THH,

HTHH, THTT,

HTHTT, THTHH.

The probability of each sequence in the first row is $\frac{1}{4}$,
each sequence in the second row is $\frac{1}{8}$,
- - - - third - $\frac{1}{16}$
fourth $\frac{1}{32}$.

The sum of these probabilities is then

$$2\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}\right) = 1-\frac{1}{16} = \frac{15}{16}.$$ This is the answer.

The number of all rearrangements of the word SAMPLING is $8!$. The number of the rearrangements where "AS" appear as a word is $7!$ (Just think of "AS" as <u>one</u> letter).

All rearrangements are equally likely. Therefore, the probability in question is $\frac{7!}{8!} = \frac{1}{8}$.

(3)

The probability that the first key opens the door is $\frac{1}{10}$.
Hence the probability that the first key does <u>not</u> open the door, and we need to pull another key, is $1-\frac{1}{10}$.

Conditioned on the event that the first key does <u>not</u> open the door, the probability that the second key opens is $1-\frac{1}{9}$
(by a similar argument, and because there are 9 keys left now).

Continue this way to the third key $\left(1-\frac{1}{8}\right)$,
fourth key $\left(1-\frac{1}{7}\right)$,
fifth key $\left(1-\frac{1}{6}\right)$.

The probability that the door will not open with one of these 5 keys =

$$\left(1-\frac{1}{10}\right)\cdot\left(1-\frac{1}{9}\right)\cdots\cdots\left(1-\frac{1}{6}\right)$$

$$=\frac{\cancel{9}}{10}\cdot\frac{\cancel{8}}{\cancel{9}}\cdot\frac{\cancel{7}}{\cancel{8}}\cdot\frac{\cancel{6}}{\cancel{7}}\cdot\frac{5}{\cancel{6}}=\frac{5}{10}=\frac{1}{2}$$

This is the answer.

IS THERE A SIMPLER WAY TO SEE THAT THE PROBABILITY IS $\frac{1}{2}$?

4

- There are $\binom{12}{2}$ ways to choose the two months.

  Now fix some choice.

- There are $2^n - 2$ ways to assign each person ~~a mont~~ one of these two months. (there are 2 unacceptable assignments, where all people get assigned ~~one or the other~~ the same month)

  Now fix some assignment.

- The probability that this assignment is true, i.e. ~~the~~ each of the n people indeed has his/her birthday in the assigned month, is $\left(\frac{1}{12}\right)^n$. (by independence).

- Therefore, the probability in question is

$$\binom{12}{2} \cdot (2^n - 2) \cdot (12)^{-n}.$$

5

The probability of missing all 10 times is $(1-p)^{10}$.
Thus the probability of hitting at least once is $1-(1-p)^{10}$.

The probability of hitting exactly once is $10(1-p)^9 p$.
(Indeed, there are 10 ways to choose the hitting shot among 10.
~~For a fixed hitting shot, t~~
Once it is fixed, the prob. of hitting is p (once), and missing all 9 remaining shots is $(1-p)^9$ (by the independence).)

Therefore, the probability of hitting at least twice is

$$1 - P(\text{no hit}) - P(\text{exactly 1 hit}) = 1 - (1-p)^{10} - 10(1-p)^9 p.$$