Interpretation of Set Theory in Probability Theory

Table

Notation	Set-theoretic interpretation	Interpretation in probability theory
ω	element or point	outcome, sample point, elementary event
Ω	set of points	sample space; certain event
$\mathscr{F}$	σ-algebra of subsets	σ-algebra of events
$A \in \mathscr{F}$	set of points	event (if $\omega \in A$, we say that event A occurs)
$\bar{A} = \Omega \setminus A$	complement of A, i.e. the set of points ω that are not in A	event that A does not occur
$A \cup B$	union of A and B, i.e. the set of points ω belonging either to A or to B	event that either A or B occurs
$A \cap B$ (or AB)	intersection of A and B, i.e. the set of points ω belonging to both A and B	event that both A and B occur
$\varnothing$	empty set	impossible event
$A \cap B = \varnothing$	A and B are disjoint	events A and B are mutually exclusive, i.e. cannot occur simultaneously
$A + B$	sum of sets, i.e. union of disjoint sets	event that one of two mutually exclusive events occurs
$A \setminus B$	difference of A and B, i.e. the set of points that belong to A but not to B	event that A occurs and B does not
$A \triangle B$	symmetric difference of sets, i.e. $(A \setminus B) \cup (B \setminus A)$	event that A or B occurs, but not both
$\bigcup_{n=1}^{\infty} A_n$	union of the sets $A_1, A_2, \ldots$	event that at least one of $A_1, A_2, \ldots$ occurs
$\sum_{n=1}^{\infty} A_n$	sum, i.e. union of pairwise disjoint sets $A_1, A_2, \ldots$	event that one of the mutually exclusive events $A_1, A_2, \ldots$ occurs
$\bigcap_{n=1}^{\infty} A_n$	intersection of $A_1, A_2, \ldots$	event that all the events $A_1, A_2, \ldots$ occur
$A_n \uparrow A$ $\left(\text{or } A = \lim_n \uparrow A_n\right)$	the increasing sequence of sets A_n converges to A, i.e. $A_1 \subseteq A_2 \subseteq \cdots$ and $A = \bigcup_{n=1}^{\infty} A_n$	the increasing sequence of events converges to event A
$A_n \downarrow A$ $\left(\text{or } A = \lim_n \downarrow A_n\right)$	the decreasing sequence of sets A_n converges to A, i.e. $A_1 \supseteq A_2 \supseteq \cdots$ and $A = \bigcap_{n=1}^{\infty} A_n$	the decreasing sequence of events converges to event A
$\overline{\lim}\, A_n$ (or $\lim\sup A_n$ or* $\{A_n \text{ i.o.}\}$)	the set $\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$	event that infinitely many of events $A_1, A_2, \ldots$ occur
$\underline{\lim}_n\, A_n$ (or $\lim\inf A_n$)	the set $\bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$	event that all the events $A_1, A_2, \ldots$ occur with the possible exception of a finite number of them

* i.o. = infinitely often.

A.N. Shiryaev, Probability, 2nd Ed. Springer, 1996. (pp. 136-137.)