Homework 10

Probability Theory – Math/Stats 525, Winter 2008.

1. Estimate the probability that the number of heads in $10,000$ independent tosses of a fair coin differs by less than 1% from $5,000$.

2. How many independent tosses of a fair coin are required for the probability that the average number of heads differs from 0.5 by less than 1% to be at least 0.99?

3. Two candidates M and O are running for President. In order to predict the outcome of the election, a number of people are selected at random and independently of the others, and they are asked their choice. The predicted percentages p_M and p_O of votes for each candidate is then worked out by calculating the portions of the selected people in the sample. How many people need to be in the sample to substantiate the following claim: *"for each candidate, the probability that the predicted percentage is correct to within* 1% *is at least* 0.95*"*?

Hint: let r_M and r_O denote the probability that a randomly chosen person will vote for M and O respectively. The claim means that $P\{|p_M - r_M| \leq 0.01\} \geq 0.95$ and $P\{|p_O - r_O| \leq 0.01\} \geq 0.95$. Use Central Limit Theorem to estimate the smallest sample size so that this holds. The estimate will depend on r_M and r_O. For which r_M and r_O do you obtain the worst (largest) estimate? A sample of this size will be used because r_M and r_O are not known in advance (the aim of the survey is to predict them).