Midterm Exam 1

Math 525, Prof. Roman Vershynin
Winter 2009 **Name:** ______________________________

Read the following information before starting the exam:

- No textbooks or note sheets are allowed on this exam. Calculators are allowed. Also allowed is one hand-written index card (3 by 5).
- The table of the distribution function $\Phi(x)$ of a standard normal random variable is included on the last page.
- Show all work, clearly and in order, if you want to get full credit. Points may be taken off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.

1. (*4 points*) An urn contains balls numbered $2, 4, 6, 8, 10$. We remove 3 balls at random (without replacement) and add up their numbers. Find the mean of the total.

2. (*4 points*) Suppose that an insurance company classifies people into one of the following three classes: low risk, average risk, and high risk. Their records indicate that the probability that a low, average, and high risk policyholder will be involved in an accident over a one-year period are 0.1, 0.2 and 0.3 respectively. Assume that 30% of the population are low risk, 50% are average risk, and 20% are high risk. What is the probability that a randomly chosen policyholder will be involved in an accident during the next year?

3. (*8 points*) Consider a random variable X with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < -1 \\ 1/2, & -1 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$$

a. (*4 pts*) Compute the moment generating function of X.

b. (*4 pts*) Derive the mean and the variance of X by differentiating the moment generating function.

4. (*12 points*) Let X and Y be independent random variables each having uniform distribution on $[0, 1]$. Let $U = X + Y$ and $V = \max(X, Y)$.

a. (*6 pts*) Compute the cumulative distribution functions of U and of V.

b. (*3 pts*) Compute the probability density functions of U and of V.

(Problem 4 contd.)

c. (*3 pts*) Compute the expectations of U and of V.

5. (*10 points*) Brad and Jeff go target shooting together. Both shoot at a target at the same time. Suppose Brad hits the target with probability $p_B = 0.7$, whereas Jeff, independently, hits the target with probability $P_J = 0.4$.

a. (*4 pts*) Determine the probability that the target is hit.

b. (*6 pts*) Given that the target is hit, what is the probability that Jeff hit it?

6. (*6 points*) An astronomer is interested in measuring the distance to a distant star. She plans to make series of measurements and then use the average value of these measurements as her estimated value of the actual distance. The astronomer believes that the values of measurements are independent and identically distributed random variables having a common mean d (the actual distance) and a common variance of 4 (light years). How many measurements does the astronomer need to make in order to be sure with probability 0.95 that her estimated distance is accurate to within ± 0.5 light year?

Bonus Question *(8 extra points, no partial credit).*

A total of n bars of magnets are placed end to end in a line with random independent orientations. Adjacent like poles repel, while adjacent poles with different polarities join to form blocks. Find the expectation of the number of blocks.