Example A of a First Exam

Answers may be expressed in terms of factorials, but not in terms of binomial or multinomial coefficients. Products do not need to be multiplied out, and fractions do not need to be written in lowest terms.

1. Suppose that 20% of birds have some red on their feathers, that 15% of birds have some blue on their feathers, that 10% of birds have some green on their feathers, that 5% have both red and green, 4% have red and blue, 3% have blue and green, and that 1% have all three. What percentage of birds have none of these colors?

2. Suppose that a sequence $\{u_n\}$ is determined by the relations $u_0 = 2$, $u_1 = 1$, and $u_{n+1} = u_n + 6u_{n-1}$ for $n = 1, 2, \ldots$. Find a (non-recursive) formula for u_n.

3. Suppose that a standard pack of 52 playing cards are dealt to four players, called North, East, South, and West, so that each player gets 13 cards. In a recent homework exercise, you were asked to use the multiplication rule of conditional probabilities to compute the probability that each player gets exactly one ace. In this problem, you are to determine this probability using only the combinatorial ideas of chapter 1.
(a) (The size of the sample space.) How many ways are there to distribute 52 distinguishable cards to 4 distinguishable players, so that each one gets 13 cards?
(b) (The size of the event.) How many ways are there to do this so that each player gets exactly one ace?
(c) Assuming that each distribution of cards is equally likely to occur, what is the probability that each placer gets exactly one ace?

4. State the three laws of probability.

5. A gambler's pocket contains two coins. One coin is fair, and the other always comes up heads. The gambler randomly withdraws a coin, and flips it.
(a) What is the probability that the coin comes up heads?
(b) Suppose now that the coin comes up heads. What is the probability that she is flipping the fair coin?
(c) Suppose that the gambler flips the coin twice, and it comes up heads both times. What is the probability that she is flipping the fair coin?
(d) Suppose that the gambler flips the coin three times, and that it comes up heads the first two times, and tails the third time. What is the probability that she is flipping the fair coin?

6. A deck of 52 cards is in random order, and the cards are turned up, one at a time.
(a) What is the probability that the fifth card turned up is an ace?
(b) What is the probability that the fifth card turned up is the first ace?
(c) Given that the fifth card turned up is the first ace, what is the probability that the next card is the ace of spades?