Example B of a First Exam

1. (a) There are 24 letters in the Greek alphabet. A fraternity name can be either two or three Greek letters, not necessaily distinct. What is the total possible number of fraternity names?
(b) How many fraternity names contain three distinct letters in Greek alphabetical order? (Hint: You might want to try the problem for smaller alphabets first.)

2. How many ways are there to arrange the letters of the word REMEMBER?

3. Suppose that E and F are events with $P\{E\} = .7$ and $P\{F\} = .4$.
(a) What is the largest possible value for $P\{E \cap F\}$?
(b) What is the smallest possible value for $P\{E \cap F\}$?

4. How many ways are there to place 16 identical balls in 4 distinguishable urns such that each urn contains at least 2 balls?

5. At a certain college, .6 of the students take a class in English, .5 take a class in French, and .3 take a class in German, .3 take both English and French, .2 take English and German, .2 take French and German, and .1 take all three languages. What fraction of the students do NOT take a class in any of the three languages?

6. The Tigers play the Red Sox in three games, and have a probability of 1/2 of winning each game; these games are independent.
(a) What is the sample space?
(b) What is the conditional probability that the Tigers win the first game, given that they win at least one game?

7. Two players take turns rolling a die. The first to roll a 6 wins the game. What is the probability that the first player wins?

8. Prove that $P\{E \cap F^C\} \geq P\{E\} - P\{F\}$ for any events E and F. State any axioms or basic results about probability that you use.

9. Urn A has 10 red balls and 5 green balls. Urn B has 8 red balls and 12 green balls. An urn (A or B) is chosen at random and then a ball is selected at random from that urn. If the ball chosen is green, what is the probability that the urn chosen was urn A?

10. Consider a weighted coin that flips heads with probability .6. Flip the coin five times. Let E be the event that the first flip is heads, and let F be the event that exactly three of the five flips are heads. Are E and F independent?

11. Two distinct cards are drawn at random from a standard 52-card deck. What is the probability that the rank of the second card is higher than or equal to the rank of the first card?

12. It is known that a certain disease occurs with probability p among patients in a clinic, and that its occurrence in different patients is independent. The clinic's lab has two ways to test blood samples: (1) it can test every sample that comes in, or (2) it can put two samples together and test them; if the two samples test positive, that is, if at least one of the samples has the disease, both must be retested individually. For what values of p does the lab expect to use fewer tests with the second strategy?