Second Sample Second Exam

1. Let X denote a random variable with $E[X] = 0$ and $\text{Var}(X) = 1$. Put $Y = 2X + 3$. (a) Find $E[Y]$. (b) Find $\text{Var}(Y)$. (c) Assume now that X is a normal variable. Find $P(Y \leq 4)$.

2. Let X denote a geometric random variable with parameter p. (a) Find $p_X(k)$ for what values of k? (b)What is $E[X]$? (c) Let a be a non-negative integer. Show that $P(X > a) = (1-p)^a$. (d) Let a and b be non-negative integers. Show that

3. Describe the random variable that would be used as a model, in the following situations. (a) A coin that comes up heads with probability p is repeatedly flipped, in independent trials, until it comes up heads for the first time. Let X denote the number of flips. Then X is a ____________________ random variable with parameter(s) ____________________.
$E[X] =$ ____________________ $\text{Var}(X) =$ ____________________

(b) The same coin as above is flipped, in independent trials, until it comes up heads for the r^{th} time. Let Y denote the number of flips. Then Y is a ____________________ random variable with parameter(s) ____________________.

$E[Y] =$ ____________________ $\text{Var}(Y) =$ ____________________

(c) A shop has on average 6 customers per hour. Let X denote the number of customers in a particular hour. Then X is a ____________________ random variable with parameter(s) ____________________.

$E[X] =$ ____________________ $\text{Var}(X) =$ ____________________

(d) In the same shop, let T denote the elapsed time (in hours) between 10:17am and the moment when the next customer enters the shop. Then T is a ____________________ random variable with parameter(s) ____________________.

$E[T] =$ ____________________ $\text{Var}(T) =$ ____________________

(e) Suppose that there are 20 passengers on a bus, and that a passenger is male with probability 0.6, independently from passenger to passenger. Let X denote the number of male passengers on the bus. Then X is a ____________________ random variable with parameter(s) ____________________.

$E[X] =$ ____________________ $\text{Var}(X) =$ ____________________

(f) Suppose that there are 110,000 football fans in the Big House, and that a fan is male with probability 0.6, independently from fan to fan. Let X denote the number of male fans in the Big House. Then the quantity $P(X > 66{,}100)$ would be estimated by using a ____________________ random variable with parameter(s) ____.

4. A fire station is to be built along a road stretching from $(0, 0)$ to $(L, 0)$ in the coordinate plane. Let $(a, 0)$ denote the position of the fire station, where $0 \leq a \leq L$. Assume that fires occur along the road at positions that are uniformly distributed from 0 to L. Let X denote the distance that the fire truck must drive to reach a fire.

(a) Find a formula for $E[X]$ in terms of a and L.
(b) How should a be chosen, in order to minimize $E[X]$?

5. The random variables X and Y take the values 0 and 1 with joint distribution $p_{X,Y}(x,y)$ as given below.

Y	1	1/3	1/3
	0	1/6	1/6
		0	1
		X	

(a) Compute the marginal statistic $p_X(x)$.
(b) Compute the marginal statistic $p_Y(y)$.
(c) Are X and Y independent? (Explain why or why not.)