Second Sample Second Exam

1. Let $X$ denote a random variable with $E[X] = 0$ and $\text{Var}(X) = 1$. Put $Y = 2X + 3$. 
   (a) Find $E[Y]$. 
   (b) Find $\text{Var}(Y)$. 
   (c) Assume now that $X$ is a normal variable. Find $P(Y \leq 4)$.

2. Let $X$ denote a geometric random variable with parameter $p$. 
   (a) Find $p_X(k)$ for what values of $k$? 
   (b) What is $E[X]$? 
   (c) Let $a$ be a non-negative integer. Show that $P(X > a) = (1 - p)^a$. 
   (d) Let $a$ and $b$ be non-negative integers. Show that

3. Describe the random variable that would be used as a model, in the following situations. 
   (a) A coin that comes up heads with probability $p$ is repeatedly flipped, in independent 
   trials, until it comes up heads for the first time. Let $X$ denote the number of flips. Then 
   $X$ is a _________________ random variable with parameter(s) _________________. 
   $E[X] =$ _______________ 
   $\text{Var}(X) = $ _________________.

   (b) The same coin as above is flipped, in independent trials, until it comes up heads for the 
   $r^{th}$ time. Let $Y$ denote the number of flips. Then $Y$ is a ________________ random 
   variable with parameter(s) _________________. 
   $E[Y] =$ _______________ 
   $\text{Var}(Y) = $ _________________.

   (c) A shop has on average 6 customers per hour. Let $X$ denote the number of customers in 
   a particular hour. Then $X$ is a ________________ random variable with parameter(s) 
   _________________. 
   $E[X] =$ _______________ 
   $\text{Var}(X) = $ _________________.

   (d) In the same shop, let $T$ denote the elapsed time (in hours) between 10:17am and the 
   moment when the next customer enters the shop. Then $T$ is a ________________ random 
   variable with parameter(s) _________________. 
   $E[T] =$ _______________ 
   $\text{Var}(T) = $ _________________.

   (e) Suppose that there are 20 passengers on a bus, and that a passenger is male with 
   probability 0.6, independently from passenger to passenger. Let $X$ denote the number of 
   male passengers on the bus. Then $X$ is a ________________ random variable with 
   parameter(s) _________________. 
   $E[X] =$ _______________ 
   $\text{Var}(X) = $ _________________.

   (f) Suppose that there are 110,000 football fans in the Big House, and that a fan is male 
   with probability 0.6, independently from fan to fan. Let $X$ denote the number of male 
   fans in the Big House. Then the quantity $P(X > 66,100)$ would be estimated by using 
   a ________________ random variable with parameter(s) _________________.

4. A fire station is to be built along a road stretching from $(0, 0)$ to $(L, 0)$ in the coordinate 
   plane. Let $(a, 0)$ denote the position of the fire station, where $0 \leq a \leq L$. Assume that 
   fires occur along the road at positions that are uniformly distributed from 0 to $L$. Let $X$ 
   denote the distance that the fire truck must drive to reach a fire.
(a) Find a formula for $E[X]$ in terms of $a$ and $L$.
(b) How should $a$ be chosen, in order to minimize $E[X]$?

5. The random variables $X$ and $Y$ take the values 0 and 1 with joint distribution $p_{X,Y}(x, y)$ as given below.

\[
\begin{array}{ccc}
Y & 0 & 1 \\
\hline
0 & 1/6 & 1/6 \\
1/3 & 1/3 & 0 \\
1 & 0 & 1 \\
\end{array}
\]

(a) Compute the marginal statistic $p_X(x)$.
(b) Compute the marginal statistic $p_Y(y)$.
(c) Are $X$ and $Y$ independent? (Explain why or why not.)