1 (15 pts)	
2 (25 pts)	
3 (30 pts)	
4 (10 pts)	
5 (20 pts)	
total (100)	

SCORE	

Exam 2

Math 425, Section 3

Instructor: Harm Derksen

(50 Minutes)

Friday, March 18, 2005

Name:

Work on the space provided. No books or notes allowed. Calculators may be used. Show your work!

Circle your final answer!

(1) A computer help-line get on average 2 calls between noon and 1pm. Let X be the number of calls between noon and 1pm on the help-line on a given day.

(a) (5 pts) What type of distribution (Bernoulli/binomial/Poisson/geometric/ negative binomial/hypergeometric/uniform) gives the best approximation for X?

Poisson, $\lambda = 2$.

(b) (10 pts) What is the probability that $X \geq 4$?

$$1 - P(0) - P(1) - P(2) - P(3) = 1 - e^{-2} - 2e^{-2} - \frac{2^2}{2}e^{-2} - \frac{2^3}{6}e^{-2} =$$

$$= 1 - \frac{19}{3}e^{-2} \approx 0.1429.$$

(2) Anna and Bert play a game. In each game, Anna pays Bert \$2.50 and she rolls two dice. If the smallest of the dice is i, then Bert pays i dollars to Anna.

(a) (15 pts) What is the expected gain of Anna?

$$(-1.5)p(-1.5)+(-0.5)p(-0.5)+(0.5)p(0.5)+(1.5)p(1.5)+(2.5)p(2.5)+(3.5)p(3.5)=$$
$$(-1.5)\frac{11}{36}+(-0.5)\frac{9}{36}+(0.5)\frac{7}{36}+(1.5)\frac{5}{36}+(2.5)\frac{3}{36}+(3.5)\frac{1}{36}=\frac{1}{36}\approx 0.0278$$

(b) (10 pts) What is the expected number of games that Anna and Bert have to play until Anna rolls at least one 1 (in which case she loses \$1.50)?

Let Y be the number of games needed. Then Y has a geometric distribution with $p = P(Anna rolls at least one 1) = \frac{11}{36}$. Therefore

$$EY = \frac{1}{\left(\frac{11}{36}\right)} = \frac{36}{11} \approx 3.273.$$

(3) X is a continuous random variable whose density function is given by

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \text{ or } x \geq 3; \\ 1/5 & \text{if } 0 \leq x < 1; \\ 2/5 & \text{if } 1 \leq x < 3. \end{cases}$$

(a) (10 pts) Determine $P(X < 2)$.

$$P(X < 2) = \int_{-\infty}^{2} f(x) = \int_{0}^{1} \frac{1}{5}\,dx + \int_{1}^{2} \frac{2}{5}\,dx = \frac{1}{5} + \frac{2}{5} = \frac{3}{5} = 0.6$$

(b) (10 pts) Compute $E(X)$.

$$EX = \int_{-\infty}^{\infty} xf(x)\,dx = \int_{0}^{1} \frac{1}{5}x\,dx + \int_{1}^{3} \frac{2}{5}x\,dx =$$
$$\frac{1}{10}x^2\Big]_0^1 + \frac{1}{5}x^2\Big]_1^3 = \frac{1}{10} + \frac{8}{5} = 1.7$$

(c) (10 pts) Compute $\mathrm{Var}(X)$.

$$EX^2 = \int_{-\infty}^{\infty} xf(x)\,dx = \int_{0}^{1} \frac{1}{5}x^2\,dx + \int_{1}^{3} \frac{2}{5}x^2\,dx =$$
$$\frac{1}{15}x^3\Big]_0^1 + \frac{2}{15}x^3\Big]_1^3 = \frac{1}{15} + \frac{52}{15} = \frac{53}{15}$$
$$\mathrm{Var}(X) = E(X^2) = (EX)^2 = \frac{53}{15} - (\frac{17}{10})^2 = \frac{193}{300} \approx 0.6433.$$

(4) (10 pts) The probability of having the rare disease *mathematosis* is 1 in 100,000. What is the probability that a city with 50,000 inhabitants has at least 2 people with this severe disease? For this question you **must** use an approximation with a Poisson random variable.

We approximate with a Poisson RV with $\lambda = np = 50,000 \cdot (1/100,000) = \frac{1}{2}$.

$$P(X \geq 2) = 1 - P(0) - P(1) = 1 - e^{-1/2} - \frac{1}{2}e^{-1/2} = 1 - \frac{3}{2}e^{-1/2} \approx 0.0902.$$

(5) (20 pts) A teacher has 10 exams in his drawer. 7 of the exams have 10 a/b multiple choice questions. The passing score for these exams is 8. The other 3 exams have 8 a/b/c multiple choice questions and have a passing score of 5. A student comes into the office. The teacher draws a random exam from his drawer. What is the probability that the student passes the exam if (s)he guesses each question?

a/b exams: binomial with $p = \frac{1}{2}$, $n = 10$:

$$P(X \geq 7) = P(7) + P(8) + P(9) + P(10) =$$

$$= \binom{10}{7}(\frac{1}{2})^{10} + \binom{10}{8}(\frac{1}{2})^{10} + \binom{10}{9}(\frac{1}{2})^{10} + \binom{10}{10}(\frac{1}{2})^{10} = \frac{11}{64} \approx 0.1719$$

a/b/c exams: binomial with $p = \frac{1}{3}$, $n = 8$:

$$P(X \geq 5) = P(5) + P(6) + P(7) + P(8) = \binom{8}{5}(\frac{1}{3})^5(\frac{2}{3})^3 + \binom{8}{6}(\frac{1}{3})^6(\frac{2}{3})^2 +$$

$$+ \binom{8}{7}(\frac{1}{3})^7(\frac{2}{3}) + \binom{8}{8}(\frac{1}{3})^8 = \frac{577}{6561} \approx 0.088$$

Bayes formula:

$$P(pass) = (0.7)(0.1719) + (0.3)(0.088) = 0.147.$$

Discrete Random Variables

distribution	parameters	$P(X=i)$	$\mathrm{E}(X)$	$\mathrm{Var}(X)$
Bernoulli	p	$p(0)=1-p, p(1)=p$	p	$p(1-p)$
Binomial	n, p	$\binom{n}{i} p^i (1-p)^{n-i}$	np	$np(1-p)$
Poisson	λ	$\frac{\lambda^i}{i!} e^{-\lambda}$	λ	λ
Geometric	p	$(1-p)^{i-1} p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative Bin.	p, r	$\binom{i-1}{r-1} p^r (1-p)^{i-r}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$
Hypergeometric	N, m, n	$\frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$	$\frac{nm}{N}$	$\frac{(N-n)nm(N-m)}{(N-1)N^2}$

Continuous Random Variables

distribution	parameters	density function	$\mathrm{E}(X)$	$\mathrm{Var}(X)$
Uniform	α, β	$f(x) = \frac{1}{\beta - \alpha}$ if $\alpha < x < \beta$, 0 otherwise	$\frac{1}{2}(\alpha+\beta)$	$\frac{1}{12}(\beta-\alpha)^2$
Normal	μ, σ	$\frac{e^{-(x-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}$	μ	σ^2