MATH 425 Winter 2012

Multiplication Principle,
1.1–1.4. Permutations, Combinations.

Lec. 1: 01/04/2012

- **Birthday Problem**: 40 students are registered for this class.

  What is the prob. that there are students who share a birthday?

  Ans: ≈ 0.89 (see later).

- \# possible birthday arrangements? $\longrightarrow$ Counting

- **Multiplication Principle**: $\left.\begin{array}{l} n \text{ ways to accomplish task } A \\ m \text{ ways } \ldots \ldots \ldots \ldots\ B \end{array}\right\} \Rightarrow nm$ ways to accomplish $A \& B$.

- Ex $\left.\begin{array}{l} AA \to \text{Ypsi} : 4 \text{ roads} \\ \text{Ypsi} \to \text{Detroit} : 10 \text{ roads} \end{array}\right\} \Rightarrow 40$ ways to get $AA \to$ Detroit.

- Ex: Flip coin 5 times. # of outcomes $= 2\cdot2\cdot2\cdot2\cdot2 = 2^5 = 32$.

  Throw die 5 times. # ... $= 6\cdot6\cdot6\cdot6\cdot6 = 6^5 = 7776$

- Def | Power set of a set $S = \{$all subsets of a set of n elements$\}$.

  $|S| = n \Rightarrow |\text{Power set}| = 2^n$. Usually denoted 2^S.

Permutations

Ex # of ways to arrange n people in a line
("# of permutations of n people")

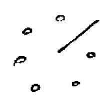
Ans: $n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1 = n!$
(by choosing 1st, 2nd, ...)

Ex: # ways to arrange n people in a line s.t. Jane is always ahead of John:

Ans: $\frac{n!}{2}$ by symmetry

Ex: # of ways to sit n people in a circle?

$\frac{n!}{n} = (n-1)!$ by symmetry.



Back to Birthday Problem:

- # of all possible birthday arrangements $= 365^{40}$.
  (# of ways to assign birthdays to the students)
- # of birthday arrangements with all different birthdays $= \underbrace{365 \cdot 364 \cdots \overset{326}{\overset{\|}{(365-39)}}}_{40} = \frac{365!}{325!}$
- All arrangements are equally likely $\Rightarrow$

  Probability that all b/d are different $= \frac{365!/325!}{365^{40}} \approx 0.11$.

  $\Rightarrow$ Prob. that some b/d are the same $= 1 - 0.11 = \boxed{0.89}$.

(Ex: Compute prob. that you share a b/d with someone else in this class $= 1 - \frac{364^{39}}{365^{39}} \approx 0.10$.)

Combinations

Ex: # of ways to choose k out of n friends to be invited for a party = ?

[Ordered selection : $\underbrace{n(n-1)\cdots(n-k+1)}_{k} = \frac{n!}{(n-k)!}$

(send postcards to 1'st, 2nd, ..

Ignore the order : $\boxed{\frac{n!}{k!(n-k)!}} =: \binom{n}{k}$.]

Def "Binomial coefficient" –

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{with convention } 0! = 1.$$

$\binom{n}{k}$ equals the # of combinations of n objects taken k at a time

= # of ways to choose k objects out of n, unordered).

(Time permitting: Ex: A child is playing with flash cards S, T, A, T, I, S, T, I, C, S

Arranges them at random in a line

P(arrangement = "STATISTICS")?

$3!$ $3!$ $2!$

[$\frac{3!\,3!\,2!}{10!} \approx 2 \cdot 10^{-5}$.]