2.1 – 2.2 Sample space and events.

lec 3: 01/09/2012

What is probability? Based on measure theory (rigorously).

- Consider an experiment whose outcome is not predictable with certainty.

Def Sample space $S = \{\text{all possible outcomes}\}$

sometimes called "elementary outcomes"

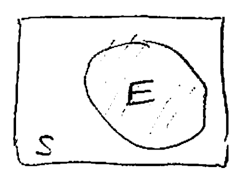
Examples:

1. Flipping a coin twice $S = \{HH, HT, TH, TT\}$ (discrete)
2. Measuring the delay of a flight. $S = [0, \infty) = \{x : x \geq 0\}$.
3. Tossing two dice $S = \{(i,j) : i,j = 1,2,3,4,5,6\}$.
4. A study is being done on all families with 1 or 2 children. Genders of the children are recorded (older first):

   $S = \{B, G, BB, BG, GB, GG\}$.
5. Location of an accident in Washtenaw County

Def An event E is any subset of a sample space S.
If an outcome of the experiment $\in E$, we say that event E occurred

Examples. (refer to Examples above)



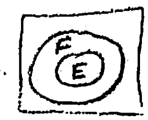
1. E = "getting head once". $E = \{HT, TH\}$
2. E = "flight departs within 1 hour of its scheduled departure"
   $E = [0, 1]$
3. E = "the sum of the two dice is at least 10.
   $E = \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}$.
4. E = "the older child is a boy". $E = \{B, BB, BG\}$.
5. E = "the accident happens within 5 miles of campus.

Operations on events

The following def. is an interpretation of the standard def of unions, intersections, etc. in probability theory.

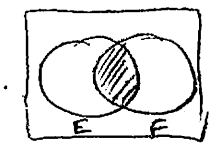
Def Let E, F be events. Venn Diagrams

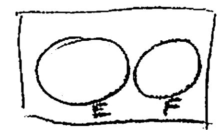
1. An event E is a subset of F if whenever E occurs, F occurs.
   $E \subset F$ or $E \subseteq F$

1. 

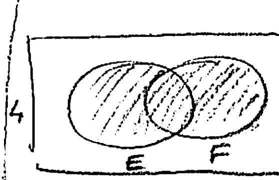
2. $E = F$ if $E \subseteq F$ and $F \subseteq E$. (E, F both occur or both don't occur).

3. Intersection $E \cap F = EF = \{$outcomes that are in both E and $F\}$
   $E \cap F$ occurs if both E AND F occur simultaneously.
   - E, F are mutually exclusive if $E \cap F = \emptyset$. (the occurrence of E prevents the occurrence of F, and vice versa)

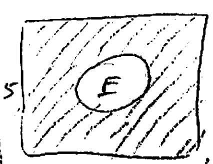
3. 



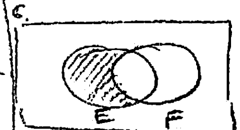
4. Union $E \cup F = \{$outcomes that are in E or F or in both$\}$.
   $E \cup F$ occurs if E OR F occurs (or both).

4 

5. Complement $E^c = \{$outcomes not in $E\}$
   E^c occurs when E does not occur

5 

6. Difference $E \setminus F = E \cap F^c = \{$outcomes that are in E but not in F"
   $E \setminus F$ occurs if E occurs but not F.

6. 

Examples.

1. $E =$ "the sum of two dice ≥ 10" (see Ex 3 p.7)
   $G =$ "the product of two dice ≥ 30" $= \{(5,6), (6,5), (6,6)\}$
   $G \subset E$

2. $E =$ "getting head once" in 2 coins (see Ex. 1 p.7)
   $F =$ "getting tail once".
   $E = F = \{HT, TH\}$.

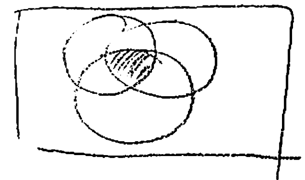
3. $E =$ "flight departs within 1 hr of its scheduled departure" (See Ex. 2 p 7)
   $F =$ "flight is delayed by at least $\frac{1}{2}$ hr"
   $E \cap F = [\frac{1}{2}, 1]$, $E^c = (1, \infty) =$ "delayed more than 1 hr", $E \setminus F = [0, \frac{1}{2})$.

4. $E =$ "two boys", $F =$ "two girls" (see Ex. 4 p.7)
   $E \cup F =$ "two children of same gender" $= \{BB, GG\}$.

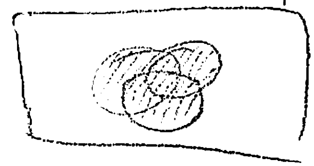
Def (Operations on multiple events)

$E_1, E_2, \ldots E_n$ events (where n may be infinite)

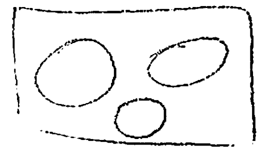
- $E_1 \cap E_2 \cap \ldots \cap E_n = \bigcap_{i=1}^{n} E_i$ = "all E_i occur"



- $E_1 \cup E_2 \cup \ldots \cup E_n = \bigcup_{i=1}^{n} E_i$ = "at least one of E_i occur"



- E_i are mutually exclusive if $E_i \cap E_j = \emptyset$ for all $i \neq j$



Ex A computer processor consists of n components. All must work in order for the processor to work.

E_i = "component i fails"

$\bigcup_{i=1}^{n} E_i$ = "some components fail" = "processor fails"

THM (De Morgan's laws) (a) $(E \cap F)^c = E^c \cup F^c$

(b) $(E \cup F)^c = E^c \cap F^c$

More generally,

(a)' $\left(\bigcap_{i=1}^{n} E_i\right)^c = \bigcup_{i=1}^{n} E_i^c$; (b)' $\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} E_i^c$

Proof (a) $(E \cap F)^c$ occurs $\Leftrightarrow$ $E \cap F$ does not occur

$\Leftrightarrow$ it can not be that both E and F occur

$\Leftrightarrow$ either E does not occur, or F does not occur, or both

$\Leftrightarrow$ either E^c occurs or F^c occurs, or both

$\Leftrightarrow$ $E^c \cup F^c$ occurs QED

Ex (for home): prove (b).

Ex (see Ex. above)

$$\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} \underbrace{E_i^c}_{i\text{'th component works}} \quad \text{(by De Morgan's law)}$$

= "all n components work" = "processor works"

Consistent with $\bigcup_{i=1}^{n} E_i$ = "processor fails"