Ex ~~Expected~~ Measurement errors ~~after~~ of an experiment are Unif(0,1)
In n indep. experiments, what is the distr. and exp. value of the <u>max</u> error?

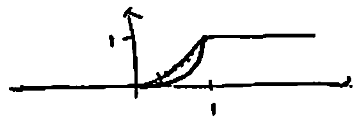
$X_1, \dots, X_n \sim \text{Unif}(0,1)$.

$X_{(n)} := \max(X_1, \dots, X_n)$

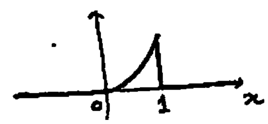
<u>CDF</u>. $F(x) = P\{X_{(n)} \le x\} = P\{\max(X_1, \dots, X_n) \le x\}$

$= P\{X_1 \le x, \dots, X_n \le x\} = P\{X_1 \le x\} \cdots P\{X_n \le x\}$ (by indep)

$$= \begin{cases} x^n, & 0 \le x \le 1. \\ 0, & x < 0 \\ 1, & x > 1 \end{cases}$$



<u>PDF</u>: $f(x) = \frac{d}{dx} F(x) = \begin{cases} n x^{n-1}, & 0 \le x \le 1 \\ 0, & \text{---} \end{cases}$



<u>Expect</u>: $E[X_{(n)}] = \int_0^1 x\, n x^{n-1}\, dx = n \int_0^1 x^n\, dx = \boxed{\frac{n}{n+1}}$ ($\to 1$ as $n \to \infty$)

~~Ex $X_{(1)} = \min(X_1, \dots, X_n)$ (!)~~.

• <u>Order statistic</u>.

<u>Remark</u> This argument is <u>general</u>, and it can be repeated for other distributions (not just Uniform)

Ex Find the distr. of <u>min</u>$(X_1, \dots, X_n)$ in this example.

6.3. Sums of indep. r.v's.

- Recall: r.v's $X_1, X_2, \dots, X_n$ indep if

$$P\{X_1\in A_1,\dots,X_n\in A_n\} = P\{X_1\in A_1\}\cdots P\{X_n\in A_n\} \quad \forall A_1 \dots A_n \subseteq \mathbb{R}$$

In discrete case $\Leftrightarrow$ $p(x_1\dots x_n)=p(x_1)\cdots p(x_n)$

in continuous case $\Leftrightarrow$ $f(x_1\dots x_n) = f(x_1)\cdots f(x_n)$.

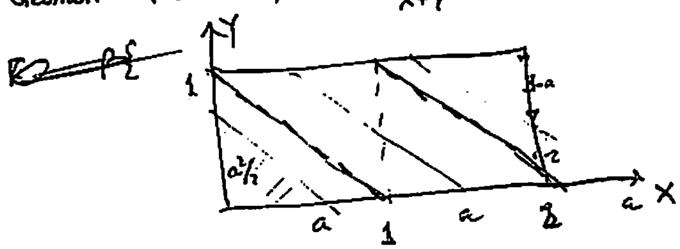
- Distribution of $X_1+\dots+X_n$? (Why important? $\bar{X} = \frac{1}{n}(X_1+\dots+X_n)$ in statistics).

Ex: $X\sim \mathrm{Unif}(0,2)$, $Y\sim \mathrm{Unif}(0,1)$ indep $X+Y\sim$?

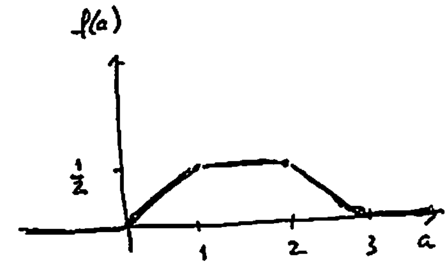
- Geometric probability:

$$F_{X+Y}(a) = P\{X+Y\le a\} = \frac{1}{2}\cdot \begin{cases} a^2/2, & 0\le a\le 1\\ \frac{1}{2} + a-1, & 1\le a\le 2\\ 2-\frac{(3-a)^2}{2}, & 2\le a\le 3.\end{cases}$$

area of rectangle



$$f_{X+Y}(a) = \frac{dF_{X+Y}(a)}{da} = \begin{cases} \frac{a}{2}, & 0\le a\le 1\\ \frac{1}{2}, & 1\le a\le 2\\ \frac{3-a}{2}, & 2\le a\le 3\\ 0, & — \end{cases} =$$



- What if not uniform (say, normal)? Geometric prob. is not applicable.