

# SOLUTIONS

Quiz 10, March 28, 2012

Introduction to Probability - MATH/STATS 425, Fall 2012

A store is about to close for the day. There are still three (unrelated) customers in the store. We estimate that it would take 10 minutes for a customer to exit the store. What is the probability that it would take more than 20 minutes for the last customer to exit the store?

(Hint: find a good model for the exit times  $T_i$  for each customer.)

$T_i \sim \text{Exp}(\lambda)$  independent

Since  $E[T_i] = \frac{1}{\lambda} = 10$ , we have  $\lambda = \frac{1}{10}$ .

The last customer exits at time  $\max(T_1, T_2, T_3)$ .

$$\begin{aligned} P\{\max(T_1, T_2, T_3) > 20\} &= 1 - P\{\max(T_1, T_2, T_3) \leq 20\} \\ &= 1 - P\{T_1 \leq 20, T_2 \leq 20, T_3 \leq 20\} \\ &= 1 - P\{T_1 \leq 20\} \cdot P\{T_2 \leq 20\} \cdot P\{T_3 \leq 20\} \\ &= 1 - (1 - e^{-\lambda \cdot 20})^3 = \boxed{1 - (1 - e^{-2})^3} \end{aligned}$$