

# SOLUTIONS

## Quiz 11, April 4, 2012

Introduction to Probability - MATH/STATS 425, Fall 2012

Let  $X$  and  $Y$  be independent exponential random variables with parameter 1. Find the joint PDF of  $U = X + Y$  and  $V = X/(X + Y)$ .

$$u = x + y, \quad v = \frac{x}{x + y}.$$

Jacobian:

$$J(x, y) = \begin{vmatrix} 1 & 1 \\ \frac{y}{(x+y)^2} & -\frac{x}{(x+y)^2} \end{vmatrix} = -\frac{x}{(x+y)^2} - \frac{y}{(x+y)^2} = -\frac{x+y}{(x+y)^2} = -\frac{1}{x+y}.$$

$$f_{X,Y}(x, y) = e^{-x} e^{-y} = e^{-(x+y)}, \quad x, y > 0.$$

$$f_{U,V}(u, v) = f_{X,Y}(x, y) |J(x, y)|^{-1} = e^{-(x+y)} (x+y) = \begin{cases} u e^{-u}, & \text{if } u > 0, 0 < v < 1 \\ 0, & \text{elsewhere.} \end{cases}$$