Solutions

Quiz 12, April 11, 2012
Introduction to Probability - MATH/STATS 425, Fall 2012

When a customer buys a computer, it is estimated that he or she will buy a computer with a Windows OS with probability \( p \), a computer with Linux OS with probability \( q \), and a computer with some other OS with probability \( r \), where of course \( p + q + r = 1 \). When \( n \) computers are sold to independent customers, compute the covariance between the number of sold computers with Windows OS and with Linux OS.

Let \( X = \) # sold computers with Windows OS,

\[ Y = \text{# sold computers with Linux OS}. \]

Then

\[ X = X_1 + \ldots + X_n, \quad Y = Y_1 + \ldots + Y_n, \]

where

\[ X_i = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ customer buys a Windows OS computer,} \\ 0, & \text{otherwise} \end{cases} \]

\[ Y_i = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ customer buys a Linux OS computer,} \\ 0, & \text{otherwise} \end{cases} \]

\[
\text{Cov}(X, Y) = \sum_{i=1}^n \sum_{j=1}^n \text{Cov}(X_i, Y_j) = \sum_{i=1}^n \text{Cov}(X_i, Y_i) \quad \text{(since } X_i, Y_i \text{ are independent)}
\]

\[
\text{Cov}(X_i, Y_i) = E [X_i Y_i] - E [X_i] E [Y_i] = -pq
\]

\[
\text{Cov}(X_i, Y_i) = \begin{cases} pq, & \text{if } X_i = 1 \text{ and } Y_i = 1 \text{(can never occur)} \\ 0, & \text{otherwise} \end{cases}
\]

Thus

\[
\text{Cov}(X, Y) = -npq.
\]