Solutions

Quiz 12, April 11, 2012
Introduction to Probability - MATH/STATS 425, Fall 2012

When a customer buys a computer, it is estimated that he or she will buy a computer with a Windows OS with probability p, a computer with Linux OS with probability q, and a computer with some other OS with probability r, where of course $p+q+r=1$. When n computers are sold to independent customers, compute the covariance between the number of sold computers with Windows OS and with Linux OS.

Let X = # sold computers with Windows OS,
Y = # sold computers with Linux OS.

Then $X = X_1 + \cdots + X_n$, $Y = Y_1 + \cdots + Y_n$,

where $X_i = \begin{cases} 1, & \text{if } i\text{'th customer buys a Windows OS computer,} \\ 0, & \text{otherwise} \end{cases}$

$Y_i = \begin{cases} 1, & \text{if } i\text{'th customer buys a Linux OS computer,} \\ 0, & \text{otherwise} \end{cases}$

$$\mathrm{Cov}(X,Y) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}(X_i, Y_j) = \sum_{i=1}^{n} \mathrm{Cov}(X_i, Y_i)$$

(since X_i, Y_j are independent for $i \neq j$, so $\mathrm{Cov}(X_i, Y_j) = 0$)

$$\mathrm{Cov}(X_i, Y_i) = \underbrace{E[X_i Y_i]}_{=0} - \underbrace{E[X_i]}_{=p}\underbrace{E[Y_i]}_{=q} = -pq$$

($X_i = 1$ and $Y_i = 1$ can never occur)

Thus $\mathrm{Cov}(X,Y) = \boxed{-npq}$