Quiz 5, February 8, 2012

Introduction to Probability - MATH/STATS 425, Winter 2012

Laura tosses 5 fair coins. If there is at most one tail, she wins \$4; otherwise she loses \$1. What is the expected Laura's winnings (losses)?

Let $X = \#$ of tails ; $X \sim \text{Binom}(5, \frac{1}{2})$

$P\{\text{at most one tail}\} = P\{X=0\} + P\{X=1\} = \binom{5}{0}\left(\frac{1}{2}\right)^5 + \binom{5}{1}\left(\frac{1}{2}\right)^5 = \frac{3}{16}$.

Let W = Laura's winnings. Takes values 4 and -1.

$P\{W=4\} = P\{\text{at most one tail}\} = \frac{3}{16}$.

$P\{W=-1\} = 1 - \frac{3}{16} = \frac{13}{16}$.

Therefore

$E[W] = 4 \cdot P\{W=4\} + (-1) \cdot P\{W=-1\} = 4 \cdot \frac{3}{16} - \frac{13}{16} = \boxed{-\frac{1}{16}}$.