Quiz 5, February 6, 2012
Introduction to Probability - MATH/STATS 425, Winter 2012

Laura tosses 5 fair coins. If there is at most one tail, she wins $4; otherwise she loses $1. What is the expected Laura's winnings (losses)?

Let \( X = \# \text{ of tails} \); \( X \sim \text{Binom}(5, \frac{1}{2}) \)

\[
p\{\text{at most one tail}\} = p\{X=0\} + p\{X=1\} = \binom{5}{0}\left(\frac{1}{2}\right)^5 + \binom{5}{1}\left(\frac{1}{2}\right)^5 = \frac{3}{16}.
\]

Let \( W = \text{Laura's winnings} \). Takes values 4 and -1.

\[
p\{W=4\} = p\{\text{at most one tail}\} = \frac{3}{16}.
\]

\[
p\{W=-1\} = 1 - \frac{3}{16} = \frac{13}{16}.
\]

Therefore

\[
E(W) = 4 \cdot p\{W=4\} + (-1) \cdot p\{W=-1\} = 4 \cdot \frac{3}{16} - \frac{13}{16} = \frac{-1}{16}.
\]